Transport Properties of Gases Obeying a Modified

Buckingham (Exp-Six) Potential*

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ABSTRACT

A large number of the Chapman and Cowling collision integrals

have been calculated for gases obeying a modified Buckingham potential,

$\varphi(r) = \dfrac{\epsilon}{1 - 6/\alpha} \left[\dfrac{6}{\alpha} e^{\alpha(1 - r/r_m)} - \left(\dfrac{r_m}{r} \right)^6 \right]$. The results are tabulated over a large

temperature range, kT/ϵ from 0 to 200, and for four values of the

parameter α , 12, 13, 14, and 15. The treatment was entirely classical,

and no corrections for quantum effects were made. The results should be

applicable to most simple, non-polar gases, and may be used to obtain informa-

tion about intermolecular forces from the observed temperature dependence of

gaseous transport properties.

The second approximation to the thermal diffusion ratio and the

third approximation to the coefficient of ordinary diffusion have been derived

according to the method of Chapman and Cowling.

Evaluation of the potential parameters for specific substances and

applications of the results will be published later.

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I. INTRODUCTION

The influence of intermolecular forces on the transport properties of gases is well known, and information concerning intermolecular forces may be obtained from the temperature variation of gaseous transport coefficients by means of the theory developed by Enskog[1] and by Chapman[2]. Such information is useful in correlating other properties of gases, as well as properties of the liquid and solid states.

The theory of Enskog and Chapman depends on the following assumptions: (1) only binary collisions between molecules are important; (2) the binary collisions are elastic; (3) the intermolecular force field is spherically symmetric; (4) molecular collisions are adequately described by classical mechanics. Thus the theory applies strictly only to monatomic gases at moderate pressures and at temperatures high enough that quantum effects are negligible. The extent to which the theoretical relations do not apply when conditions (2) and (3) are not met is not yet known[3]. However, the theory has had success in correlating transport phenomena in polyatomic gases, so that these conditions may not be a severe limitation[4].

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[1] D. Enskog, Phys. Zeit. 12, 56, 533 (1911); Inaug. Diss. Upsala (1917).

[2] S. Chapman, Phil. Trans. Roy. Soc. (London) A211, 433 (1912); A216, 279 (1916); A217, 115 (1917).

[3] In the case of the thermal conductivity it is known that the Enskog-Chapman theory fails badly when energy may be transported by means of the molecular internal degrees of freedom.

[4] S. Chapman and T. G. Cowling, The Mathematical Theory of Non-Uniform Gases (Cambridge University Press, Teddington, England, 2nd edition, 1952), p. 7.

In order to obtain specific information about intermolecular forces from transport phenomena, it is necessary to assume an analytical form for the intermolecular potential, calculate from this the Enskog-Chapman "collision integrals", or "temperature-dependent cross sections", as a function of temperature and the potential parameters, and evaluate the parameters for any specified gas by comparison with collision integrals derived from experimental measurements. If the theory is correct and the chosen potential form suitable, a single set of potential parameters should suffice to describe all the transport properties of the gas, as well as other properties, such as the equation of state. Except for very simple and physically unrealistic potential forms, the calculation of the collision integrals involves extensive numerical integrations.

The best intermolecular potential used to date for the study of transport phenomena is of the Lennard-Jones form, with a repulsion term varying as the inverse twelfth power of the distance of separation between centers of two molecules, and an attraction term varying as the inverse sixth power of the separation distance. The collision integrals for this potential have been evaluated independently by a number of workers[5]-[8], and their results have

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[5] T. Kihara and M. Kotani, Proc. Phys.-Math. Soc. Japan 25, 602 (1943).

[6] J. de Boer and J. van Kranendonk, Physica 14, 442 (1948).

[7] Hirschfelder, Bird, and Spotz, J. Chem. Phys. 16, 968 (1948); 1343 (1949).

[8] J. S. Rowlinson, J. Chem. Phys. 17, 101 (1949).

been extensively compared with experiment[9]-[14]. The results obtained

have not been entirely satisfactory, and the question arises whether this

is primarily the fault of the theory or of an inadequate potential form. That

the Lennard-Jones (12-6) potential is not entirely adequate is shown by lack

of complete agreement between theory and experiment even for the rare gases.

It is the purpose of the present article to describe calculations of

the Enskog-Chapman collision integrals for a more realistic potential form

than has been used previously. It is hoped that some light may be thrown on

the reasons for the present lack of complete agreement between theory and

experiment. The inverse sixth power term in the Lennard-Jones (12-6)

potential represents the leading term in the theoretical form for the dispersion

energy, but the inverse twelfth power is only an empirical approximation to

the repulsion energy, and is not expected to be accurate over a large range of

separation distance. There is a considerable amount of theoretical and

experimental evidence which indicates that the repulsion energy is more

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(9)a Hirschfelder, Bird, and Spotz, Chem. Rev. 44, 205 (1949); Trans. Am.
Soc. Mech. Engrs. 71, 921 (1949).

b Hirschfelder, Curtiss, Bird, and Spotz, The Molecular Theory of Gases
and Liquids (John Wiley and Sons, New York, 1954), Chapter 8.

(10) K. E. Grew, Proc. Phys. Soc. (London) 62, 655 (1949); K. E. Grew and
T. L. Ibbs, Thermal Diffusion in Gases (Cambridge University Press,
Teddington, England, 1952).

(11) E. R. S. Winter, Trans. Faraday Soc. 46, 81 (1950).

(12) E. Whalley and W. G. Schneider, J. Chem. Phys. 20, 657 (1952).

(13) B. N. Srivistava and M. P. Madan, Phil. Mag. 43, 968 (1952); Proc.
Phys. Soc. (London) A66, 277 (1953).

(14) Amdur, Ross, and Mason, J. Chem. Phys. 20, 1620 (1952).

suitably described by an exponential form[15]-[20]. Accordingly we have

chosen the form

$$\varphi(r) = \frac{\epsilon}{1 - 6/\alpha} \left[\frac{6}{\alpha} e^{\alpha(1 - r/r_m)} - \left(\frac{r_m}{r} \right)^6 \right] , \tag{1}$$

where $\varphi(r)$ is the potential energy of the molecules at separation distance

r , $-\epsilon$ is the minimum potential energy, r_m is the value of r for which

$\varphi(r)$ is a minimum, and α is an additional parameter which may be considered

a measure of the steepness of the repulsion energy. The next higher term in the

dispersion energy, a term varying as r^{-8} and representing the dipole-quadrupole

energy, has not been included since the same effect as adding such a term may

be achieved to a good approximation by a variation of the parameter α .

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[15] C. Zener, Phys. Rev. 37, 556 (1931).

[16] R. H. Fowler and E. A. Guggenheim, Statistical Thermodynamics (Cambridge University Press, Teddington, England, 1939), pp. 276-279, 291-294, give a summary of the evidence prior to 1939.

[17] R. A. Buckingham and J. Corner, Proc. Roy. Soc. (London) A189, 118 (1947); J. Corner, Trans. Faraday Soc. 44, 914 (1948).

[18] I. Amdur, J. Chem. Phys. 17, 844 (1949).

[19] J. L. Yntema and W. G. Schneider, J. Chem. Phys. 18, 646 (1950).

[20] M. Kunimune, J. Chem. Phys. 18, 754 (1950); Prog. Theor. Phys. 5, 412 (1950).

A typical potential energy curve is shown in Figure 1. A more easily visualized measure of the steepness of the repulsion energy than α is the ratio σ/r_m , where σ is the value of r for which $\varphi(r) = 0$. Table I gives σ/r_m as a function of α ; included for comparison is σ/r_m for the Lennard Jones (12-6) potential.

Table I. Ratio of the low velocity collision diameter, σ , to the position of the minimum, r_m .

α	σ/r_m
12	0.8761
13	0.8832
14	0.8891
15	0.8942
Lennard-Jones (12-6)	0.8909

A potential of the form of Eq. (1) was derived theoretically by Slater and Kirkwood[21] for the special case of helium, but its use as a semi-empirical form and its extensive application has been carried out mainly by Buckingham[22].

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[21] J. C. Slater, Phys. Rev. **32**, 349 (1928); J. C. Slater and J. G. Kirkwood, Phys. Rev. **37**, 682 (1931).

[22] R. A. Buckingham, Proc. Roy. Soc. (London) A168, 264 (1938); H. S. W. Massey and R. A. Buckingham, ibid. A168, 378 (1938); A169, 205 (1938); Buckingham, Hamilton, and Massey, ibid. A179, 103 (1941); R. A. Buckingham and R. A. Scriven, Proc. Phys. Soc. (London) **65**, 376 (1952).
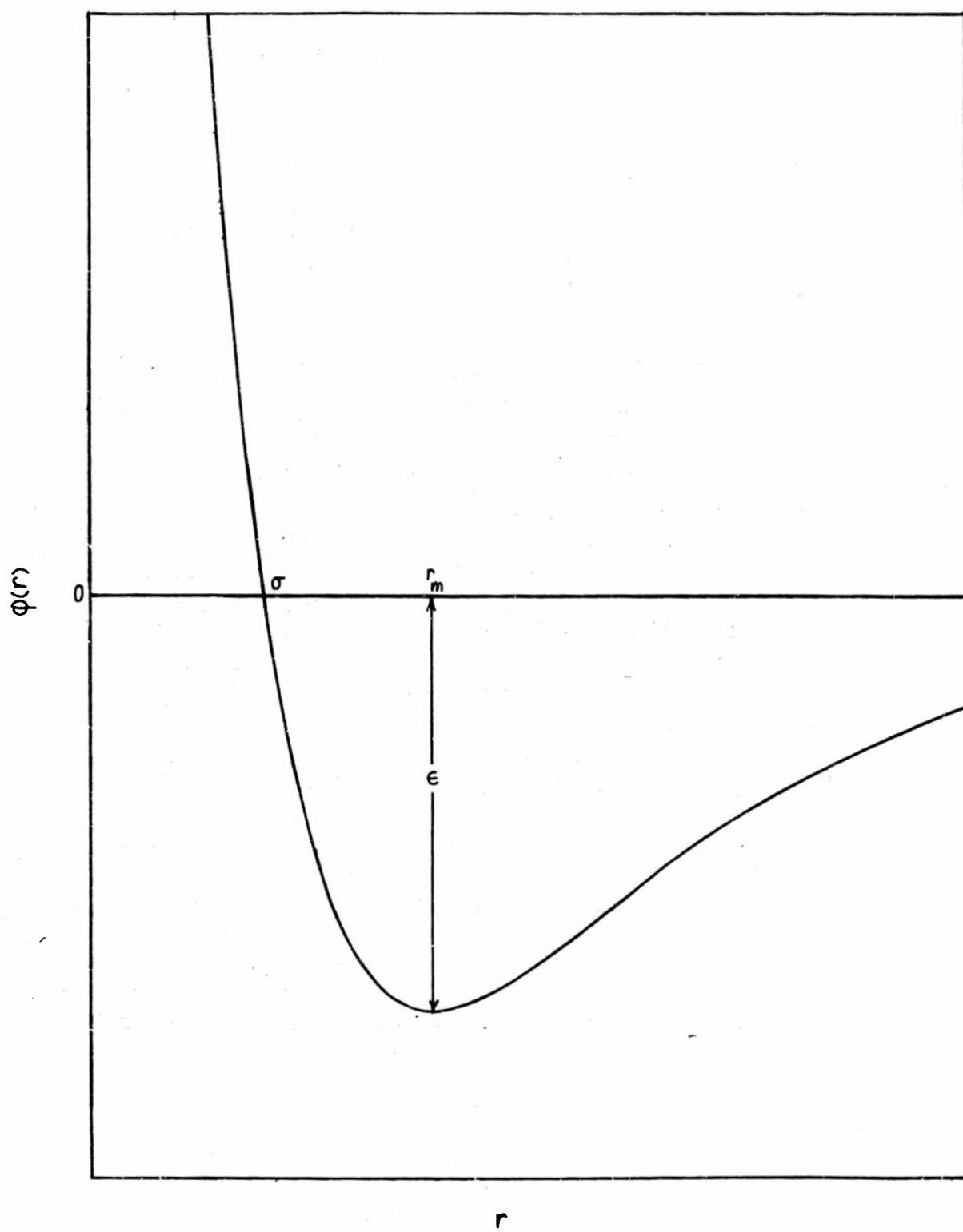
Fig. 1. A typical intermolecular potential energy curve.

This potential has the defect that at a small separation distance, r_{max} , it has a/maximum, and approaches minus infinity as r approaches zero. There-
fore Buckingham usually has used Eq. (1) only for $r > r_m$, and for $r < r_m$
used another form in which the inverse sixth power term of Eq. (1) was

multiplied by another exponential term to prevent the potential from having

a maximum. However, the maximum occurs at such high energies that it has

little effect on ordinary thermal collisions, and we believe it is preferable

to use Eq. (1) for $r > r_{max}$, and to set the potential equal to infinity for

$r \leq r_{max}$. We have called this form a modified Buckingham potential, which

might be referred to as the "Exp-Six" potential.

II. GENERAL FORMULAS

According to the theory of Enskog and Chapman, the transport

phenomena depend on the intermolecular potential through collisions of single

pairs of molecules, and the only feature of a collision which it is necessary to

consider is the angle of deflection, χ , through which the relative velocity

vector of a pair of molecules under consideration is rotated by the collision.

The angle χ is given by[23][24]

$$\chi(v,b) = \pi - 2b \int_{r_c}^{\infty} \left[1 - b^2/r^2 - 2\varphi(r)/\mu v^2 \right]^{-\frac{1}{2}} dr/r^2 , \qquad (2)$$

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[23] E. H. Kennard, Kinetic Theory of Gases (McGraw-Hill Book Co., Inc. New York, 1938), pp. 115-122.

[24] Reference (9b), Chapter 1.

where v is the absolute value of the relative initial velocity of the pair of molecules, b , the collision parameter, is the perpendicular distance between one of the molecules and the initial line of relative approach of the other, μ is the reduced mass of the colliding pair, and r_c is the distance of closest approach during the collision, given by

$$1 - b^2/r_c^2 - 2\varphi(r_c)/\mu v^2 = 0 \; . \tag{3}$$

The reduced velocity-dependent cross sections are determined from χ by

$$S^{(l)}(K) = 2\left[1 - \frac{1}{2} \frac{1+(-1)^l}{1+l}\right]^{-1} \int_0^\infty (1 - \cos^l \chi) \beta \, d\beta \; , \tag{4}$$

where we have introduced the reduced quantities $K = \mu v^2/2\epsilon$ and $\beta = b/r_m$. Finally, the reduced collision integrals, which directly determine the transport properties, are

$$\Omega^{(l,n)*}(T)^* = 2\left[(n+1)!(T^*)^{n+2}\right]^{-1} \int_0^\infty e^{-K/T^*} K^{n+1} S^{(l)}(K) \, dK, \tag{5}$$

where $T^* = kT/\epsilon$, k is Boltzmann's constant, and T is the absolute temperature. The cross sections have been reduced by dividing by πr_m^2, and the collision integrals are just the $\Omega^{(l)}(n)$ of Chapman and Cowling[25], divided by their values for rigid spheres of diameter r_m. Thus $S^{(l)}(K)$ and $\Omega^{(l,n)*}(T^*)$ are unity for rigid spheres.

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[25] Reference (4), pp. 157, 160.

The values of the transport coefficients may be expressed as infinite series, and higher approximations to the coefficients are obtained the more terms of the series that are taken. Fortunately, the convergence is rapid, and very few terms are needed. The coefficient of viscosity, η , of a pure gas is

$$\eta = \frac{5}{16} \left(\frac{MRT}{\pi} \right)^{\frac{1}{2}} \frac{f_\eta}{r_m^2 \, \Omega^{(2,2)*}(T^*)} \quad , \tag{6}$$

where M is the molecular weight, R is the gas constant per mole, and f_η represents the infinite series, and is a complicated function of the collision integrals. The term f_η is nearly unity, and is a slowly varying function of T^* ; in Appendix I the expression for the first three terms of f_η is given in terms of the collision integrals.

The coefficient of thermal conductivity, λ , of a pure gas is

$$\lambda = \frac{25}{32} \left(\frac{RT}{\pi M} \right)^{\frac{1}{2}} \frac{C_v}{r_m^2} \frac{f_\lambda}{\Omega^{(2,2)*}(T^*)} \quad , \tag{7}$$

where C_v is the molar heat capacity at constant volume of the gas, and f_λ is a term similar to f_η . The first three terms of f_λ are given in terms of the collision integrals in Appendix I. The formula (7) does not take account of the large amount of energy transported by the internal degrees of freedom of the molecules, and thus applies only to monatomic gases. An approximate correction for this extra energy transport may be made by multiplying the

right hand side of Eq. (7) by the "Eucken factor"[26][27][28], which is usually taken to be $(9 \gamma /10 - 1/2)$, where γ is the ratio of the constant pressure heat capacity to the constant volume heat capacity of the gas. No exact method for making this correction has yet been worked out.

The coefficient of mutual diffusion of a binary mixture of gases of types 1 and 2 is

$$D_{12} = \frac{3}{16} \left(\frac{M_1 + M_2}{M_1 M_2} \quad \frac{2RT}{\pi} \right)^{\frac{1}{2}} \frac{f_D}{n \ (r_m)_{12}^2 \ \Omega_{12}^{(1,1)*}(T^*)} , \quad (8)$$

where D_{12} is the mutual diffusion coefficient, M_1 and M_2 are the molecular weights of the species 1 and 2, and n is the total molecular density. The subscripts 1, 2 refer to an interaction between a molecule of type 1 and one of type 2. The term f_D is a very complicated function not only of T^*, but also of M_1, M_2, and the composition of the diffusing mixture. The dependence of D_{12} on the composition is entirely contained in this term, as well as the dependence of D_{12} on 1-1 and 2-2 molecular interactions. The third approximation to f_D is given in Appendix I. It seemed worthwhile to extend the expressions for the diffusion and thermal diffusion coefficients to one more

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[26] A. Eucken, Phys. Zeit. 14, 324 (1913).

[27] Reference (4), pp. 237-240.

[28] Reference (9b), Chapter 7.

approximation than has previously been used for a realistic intermolecular potential, inasmuch as there is evidence[29] that the lower approximations might differ from the limiting theoretical values by amounts greater than the error involved in determining the quantities experimentally. The expression :or the self-diffusion coefficient, D_{11}, is obtained from Eq. (8) by setting $M_1 = M_2$ and deleting the subscripts 1, 2.

The general expression for the thermal diffusion ratio is quite complicated, and for computational purposes is perhaps best expressed in the determinant notation of Chapman and Cowling. In this form, the expression for the m-th approximation to the thermal diffusion ratio, $\left[k_T\right]_m$, of a binary mixture is

$$\left[k_T\right]_m = \frac{5}{2} \frac{x_1 \mathscr{A}_{01}^{(m)} \left(\frac{M_1 + M_2}{2M_1}\right)^{1/2} + x_2 \mathscr{A}_{0-1}^{(m)} \left(\frac{M_1 + M_2}{2M_2}\right)^{1/2}}{\mathscr{A}_{00}^{(m)}} , \quad (9)$$

where x_1 and x_2 are the mole fractions of the two components. $\mathscr{A}^{(m)}$ is a determinant of $(2m + 1)$ order, whose general term is a_{ij}, where i and j range from $-m$ to m. $\mathscr{A}_{ij}^{(m)}$ is the minor of $\mathscr{A}^{(m)}$ obtained by striking out the row and col.nn containing a_{ij}. The elements a_{ij} are functions of the collision integrals $\Omega^{(\ell, n)*}$, the molecular weights of the component gases, and the composition of the mixture. The expressions for the a_{ij} for $m = 2$ are given in Appendix I.

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[29] Reference (4), pp. 169, 196.

For binary mixtures of heavy isotopes, where the forces between

the different molecular species are the same, Eq. (9) may be put into a

simpler form by expanding the determinants in powers of the small quantity

$(M_1 - M_2)/(M_1 + M_2)$, and keeping only the first power.   To this order of

approximation,

$$[k_T]_m = [k_T^*]_m \left(\frac{M_1 - M_2}{M_1 + M_2} \right) x_1 x_2 \quad , $$

$$(10)$$

where $[k_T^*]_m$ is a reduced thermal diffusion ratio, and is a function of the

reduced temperature alone.   Since the expression for $[k_T^*]_m$ in terms of

the collision integrals is complicated, it is placed in Appendix I.   We prefer

the function k_T^* to the more frequently encountered R_T , for reasons which

are discussed in detail at the end of Appendix I.   The two functions are very

simply related: $R_T = (118/105) \ k_T^*$.

For mixtures of gases, the following three ratios of collision

integrals occur frequently:

$$A^* = \Omega_{12}^{(2,2)*} \Big/ \Omega_{12}^{(1,1)*} \quad , $$

$$B^* = \left[5\Omega_{12}^{(1,2)*} - 4\Omega_{12}^{(1,3)*} \right] \Big/ \Omega_{12}^{(1,1)*} \quad , \qquad (11)$$

$$C^* = \Omega_{12}^{(1,2)*} \Big/ \Omega_{12}^{(1,1)*} \quad . $$

These quantities are equal to unity for rigid spheres.

III. NUMERICAL CALCULATIONS

The first step in the evaluation of the collision integrals is the calculation by numerical integration of the angle of deflection, χ, as a function of v, b, and the potential parameters. Eq. (2) is inconvenient for numerical computations because the integrand becomes infinite at the lower limit of the integral. This singularity may be removed by substituting into Eq. (2) the value of b from Eq. (3), and making the change of variable $\sin \theta = r_c / r$, after which Eq. (2) may be rearranged into the form

$$\chi = \pi - \frac{2\beta}{y} \int_0^{\frac{\pi}{2}} \left[1 - \frac{\sin^2\theta + \sin^4\theta}{K(1-6/\alpha) y^6} + \frac{(6/\alpha)e^\alpha}{K(1-6/\alpha)} \left(\frac{e^{-\alpha y}\sin^2\theta - e^{-\alpha y/\sin\theta}}{\cos^2\theta} \right) \right]^{-\frac{1}{2}} d\theta, \tag{12}$$

where y is the reduced distance of closest approach, r_c/r_m, and β is the reduced collision parameter, b/r_m, given by

$$\beta = y \left[1 - \frac{1}{K(1-\frac{6}{\alpha})} \left(\frac{6}{\alpha} e^{\alpha(1-y)} - \frac{1}{y^6} \right) \right]^{\frac{1}{2}}. \tag{13}$$

The integration in Eq. (12) was performed numerically, the integrand being evaluated at intervals of $\theta = \pi/60$ in cases where the integrand varied rapidly with θ, but otherwise at intervals of $\theta = \pi/20$ or $\pi/12$, depending on the behavior of the integrand. The integrand in Eq. (12) becomes indeterminate at $\theta = \pi/2$, but may be shown by L'Hospital's Rule to approach the limit

$$\left[1 - \frac{2}{K(1-\frac{6}{\alpha}) y^6} + \frac{\frac{6}{\alpha} e^{\alpha(1-y)}}{K(1-\frac{6}{\alpha})} \left(\frac{\alpha y}{2} - 1 \right) \right]^{-\frac{1}{2}}. \tag{14}$$

The numerical integration of Eq. (12) was sufficient to determine χ to a few ten-thousandths of a radian. Values of χ , together with the corresponding values of β , have been tabulated as a function of y and K for α = 12, 13, 14, and 15; the length of the tabulations prevented their publication in the present paper[30]. The tabulations cover thirteen values of K for each value of α , as in Table III, and an average of about 20 values of y for each K . The size of the intervals on y varies from 0.005 to 0.40, depending on the behavior of χ , and was chosen so as to obtain an accuracy of 0.1 percent in the integration for $S^{(l)}(K)$.

The calculation of χ was complicated by the fact that for some collisions which occur at less than a certain critical energy, K_o , the molecules may "orbit" around each other for an indefinite number of revolutions, and χ consequently tends to minus infinity. Such orbiting collisions have been discussed in connection with the Lennard-Jones (12-6) potential[7][14], and the same general behavior is observed with the present potential. For energies equal to or less than K_o , there is a critical value of beta, β_c , for which orbiting will occur. There are two values of the distance of closest approach, y , which correspond to β_o ; values of y between these two critical ones cannot occur physically. The larger y , designated as y_o , will cause the integrand of Eq. (12) to become infinite at the upper limit, so that the value of y_o may be determined by numerical

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[30] Multigraphed tables may be obtained from the author on request.

solution of the equation

$$1 - \frac{2}{K(1-6/\alpha) y_o^6} + \frac{(6/\alpha) e^{\alpha(1-y_o)}}{K(1-6/\alpha)} \left(\frac{\alpha y_o}{2} - 1 \right) = 0 , \quad (15)$$

which is obtained from Eqs. (12) and (14). The value of β_o may

be found by substituting the value of y_o into Eq. (13) and solving. The

smaller value of y , designated as y_1 , also satisfies Eq. (13) with $\beta = \beta_o$,

and may be calculated in this way. At $y = y_1$ the integrand of Eq. (12) becomes

infinite for $\varphi = \arcsin(y_1/y_o)$. Between $y = y_o$ and $y = y_1$ there are two

values of y which give the same β when substituted into Eq. (13), but

these values are without physical significance. Values of β_o , y_o , and y_1

for several values of K are given in Table II; y_o and y_1 become equal at

$K = K_o$.

For small values of χ , it is useful to have the asymptotic

series, valid for large values of y :

$$\chi = - \frac{15\pi}{16} \left[\frac{1}{K(1-6/\alpha) y^6} \right] - \frac{585\pi}{1024} \left[\frac{1}{K(1-6/\alpha) y^6} \right]^2$$

$$+ \frac{(6/\alpha) e^{\alpha(1-y)}}{K(1-6/\alpha)} \left(\frac{\pi \alpha y}{2} \right)^{\frac{1}{2}} \left(1 - \frac{1}{8\alpha y} + \frac{9}{32(\alpha y)^2} - \cdots \right) , \quad (16)$$

which was obtained by the method of Amdur and Pearlman[31].

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[31] I. Amdur and H. Pearlman, J. Chem. Phys. 9, 503 (1941);
I. Amdur, J. Chem. Phys. 15, 482 (1947).

Table II. Conditions for orbiting.

α	K	β_0	y_0	y_1
12	.4	1.7916	1.4468	.9694
	$K_0 = .78101$	1.5835	1.1849	1.1849
13	.4	1.7716	1.4353	.9649
	.8	1.5636	1.2098	1.1076
	$K_0 = .82691$	1.5530	1.1708	1.1708
14	.4	1.7555	1.4253	.9621
	.8	1.5530	1.2209	1.0710
	$K_0 = .86809$	1.5281	1.1594	1.1594
15	.4	1.7421	1.4166	.9604
	.8	1.5437	1.2248	1.0504
	$K_0 = .90530$	1.5075	1.1498	1.1498

To evaluate the cross sections $S^{(\ell)}(K)$ defined in Eq. (4), the integration variable was changed from β to y , and Eq. (4) transformed to

$$S^{(\ell)}(K) = \frac{2}{\left[1 - \frac{1}{2}\frac{1+(-1)^{\ell}}{1+\ell}\right]} \int_{y_c}^{\infty} (1-\cos^{\ell}\chi)\left[1 - \frac{2}{K(1-6/\alpha)y^6} + \frac{(6/\alpha)e^{\alpha(1-y)}}{K(1-6/\alpha)}\left(\frac{\alpha y}{2} - 1\right)\right] y\, dy,$$

(17)

where y_c is the value of y for which $\beta = 0$, and represents the reduced distance of closest approach for a head-on collision. Most of the integration could then be carried out numerically, using the values of χ tabulated as a function of y for a given K and α . To avoid the necessity of calculating accurate values of y_c from Eq. (13), the numerical integration was started from a lower limit y_A which was slightly greater than y_c , and the integration from y_c to y_A executed as follows. Eqs. (12) and (13) were differentiated to form $(d\chi/d\beta)$ in terms of χ and β for the limit of $\beta \longrightarrow 0$. Solution of the resulting differential equation then showed that χ varied linearly with β when β was near zero, so that the integration from y_c to y_A could be performed analytically. The equations obtained for these small contributions to the cross sections are:

$$A^{(1)}(K) = \beta_A^2 - \frac{2\beta_A^2}{(\pi-\chi_A)^2}\left[1 - \cos(\pi-\chi_A)\right] + \frac{2\beta_A^2}{\pi-\chi_A}\sin(\pi-\chi_A),$$

$$A^{(2)}(K) = \frac{3}{4}\beta_A^2 + \frac{3}{8}\frac{\beta_A^2}{(\pi-\chi_A)^2}\left[1 - \cos 2(\pi-\chi_A)\right] - \frac{3}{4}\frac{\beta_A^2}{\pi-\chi_A}\sin 2(\pi-\chi_A),$$

(18)

$$A^{(3)}(K) = \beta_A^2 - \frac{1}{18} \frac{\beta_A^2}{(\pi - \chi_A)^2} \left[1 - \cos 3(\pi - \chi_A) \right] - \frac{3}{2} \frac{\beta_A^2}{(\pi - \chi_A)^2} \left[1 - \cos(\pi - \chi_A) \right]$$

$$+ \frac{1}{6} \frac{\beta_A^2}{\pi - \chi_A} \sin 3(\pi - \chi_A) + \frac{3}{2} \frac{\beta_A^2}{\pi - \chi_A} \sin(\pi - \chi_A) ,$$

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$$A^{(4)}(K) = \frac{25}{32} \beta_A^2 + \frac{5}{256} \frac{\beta_A^2}{(\pi - \chi_A)^2} \left[1 - \cos 4(\pi - \chi_A) \right]$$

$$+ \frac{5}{16} \frac{\beta_A^2}{(\pi - \chi_A)^2} \left[1 - \cos 2(\pi - \chi_A) \right] - \frac{5}{64} \frac{\beta_A^2}{\pi - \chi_A} \sin 4(\pi - \chi_A)$$

$$- \frac{5}{8} \frac{\beta_A^2}{\pi - \chi_A} \sin 2(\pi - \chi_A) ,$$

(18)
(cont.)

where the $A^{(\ell)}(K)$ are the contributions to the $S^{(\ell)}(K)$ for the range y_C to y_A, and β_A and χ_A are the values of β and χ corresponding to y_A. The numerical integration of Eq. (17) when no orbiting occurred $(K > K_o)$ was carried out from y_A to y_E, where y_E was a large value of y, and the remainder of the integration, from y_E to infinity, carried out by substituting the small angle formula, Eq. (16), into Eq. (17), expanding $(1 - \cos^\ell \chi)$, and integrating analytically to yield:

$$\frac{1}{\ell} \left[1 - \frac{1}{2} \frac{1 + (-1)^\ell}{1 + \ell} \right] F^{(\ell)}(K) = \left(\frac{15\pi}{16} \right)^2 y_E^2 \left[\frac{1}{10} \left(\frac{1}{K(1 - 6/\alpha)y_E^6} \right)^2 - \frac{25}{512} \left(\frac{1}{K(1 - 6/\alpha)y_E^6} \right)^3 \right]$$

$$- \left(\frac{15\pi}{16} \right) \frac{(6/\alpha) e^{\alpha(1 - y_E)}}{K(1 - 6/\alpha)} \left(\frac{2\pi}{\alpha y_E^3} \right)^{\frac{1}{2}} \left[1 - \frac{37}{8\alpha y_E} + \frac{823}{32(\alpha y_E)^2} - \cdots \right] ,$$

(19)

where $F^{(\ell)}(K)$ is the contribution to $S^{(\ell)}(K)$ for the range y_E to infinity.

When K is equal to or less than the critical energy for orbiting, K_o, numerical integration of Eq. (17) cannot be carried out over the entire range from y_A to y_E because: (1) the region from y_1 to y_o is physically inaccessible, and in fact χ is imaginary there; (2) the integrand oscillates violently near y_1 and y_o, where χ goes to minus infinity. For $K \leq K_o$ the integral from y_A to y_E was therefore broken up into four integrals: $B^{(\ell)}(K)$, $C^{(\ell)}(K)$, $D^{(\ell)}(K)$, and $E^{(\ell)}(K)$. The integral $B^{(\ell)}(K)$ extended from y_A to y_B, where y_B was slightly less than y_1, and was evaluated numerically. Similarly, the integral $E^{(\ell)}(K)$ extended from y_D, where y_D was slightly greater than y_o, to y_E, and was also evaluated numerically. The integrals $C^{(\ell)}(K)$ and $D^{(\ell)}(K)$, covering the regions y_B to y_1 and y_o to y_D, respectively, were calculated by the method of Hirschfelder, Bird, and Spotz[7], in which χ is represented near the regions of orbiting by equations of the form

$$\chi^2 = \text{const.}/(\beta^2 - \beta_o^2) ,$$

(20)

and Eq. (4) is integrated analytically between the proper limits. The results of this integration are:

$$C^{(1)}(K) = (\beta_o^2 - \beta_B^2)\left[1 - \cos \chi_B + \chi_B \sin \chi_B - \chi_B^2 Ci(-\chi_B) \right] , \qquad 20$$

$$C^{(2)}(K) = \frac{3}{4}(\beta_o^2 - \beta_B^2)\left[1 - \cos 2\chi_B + 2\chi_B \sin 2\chi_B - 4\chi_B^2 Ci(-2\chi_B) \right] ,$$

$$C^{(3)}(K) = \frac{1}{4}(\beta_o^2 - \beta_B^2)\left[\begin{array}{l} 4 - \cos 3\chi_B - 3\cos \chi_B + 3\chi_B \sin 3\chi_B \\[2mm] + 3\chi_B \sin \chi_B - 9\chi_B^2 Ci(-3\chi_B) - 3\chi_B^2 Ci(-\chi_B) \end{array} \right] ,$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (21)$$

$$C^{(4)}(K) = \frac{5}{32}(\beta_o^2 - \beta_B^2)\left[\begin{array}{l} 5 - \cos 4\chi_B - 4\cos 2\chi_B + 4\chi_B \sin 4\chi_B \\[2mm] + 8\chi_B \sin 2\chi_B - 16\chi_B^2 Ci(-4\chi_B) \\[2mm] - 16\chi_B^2 Ci(-2\chi_B) \end{array} \right] ,$$

where β_B and χ_B are the values of β and χ corresponding to y_B , and $Ci(-\chi_B)$ is the cosine integral, a tabulated function[32], defined as follows:

$$Ci(z) = \int_\infty^z \frac{\cos t}{t} \, dt . \qquad\qquad\qquad (21b)$$

The $D^{(\ell)}(K)$ are of the same form as the $C^{(\ell)}(K)$.

The cross sections $S^{(\ell)}(K)$ are not convenient to use directly, since they vary rapidly when K is small, and tend to infinity as K goes to zero. Kihara and Kotani have shown[5] that in the limit $K \to 0$, the cross

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[32]
Federal Works Agency, Work Projects Administration for the City of New York, Tables of Sine, Cosine, and Exponential Integrals, Vol. II (1940); Table of Sine and Cosine Integrals (1942).

sections are independent of the potential energy of repulsion, and depend on the attraction energy alone. From this it is easy to show that the $S^{(\ell)}(K)$ are proportional to $\left[K(1 - 6/\alpha) \right]^{-1/3}$ for $K \to 0$; we therefore define the functions $R^{(\ell)}(K)$:

$$R^{(\ell)}(K) = \left[K(1 - 6/\alpha) \right]^{1/3} S^{(\ell)}(K), \tag{22}$$

which are given in Table III, and plotted for $\alpha = 12$ and 15 in Figure 2 for the two most important cases, $\ell = 1$ and 2. For comparison, the analogous functions for the Lennard-Jones (12-6) potential are also plotted. Since $R^{(\ell)}(0)$ is independent of the repulsion, the values given in the table for $\ell = 1$ and 2 were taken directly from Kihara and Kotani[5]. For $\ell = 3$ and 4, the values of $R^{(\ell)}(0)$ were obtained by extrapolation, a procedure which is permissible because these functions occur in the expressions for the transport properties only as correction terms.

For use in evaluating the collision integrals, the cross section functions were approximated by the following algebraic expressions:

$$R^{(\ell)}(K) = R^{(\ell)}(0) \left(1 + a_1^{(\ell)} K^{\frac{1}{3}} + a_2^{(\ell)} K^{\frac{2}{3}} \right), \qquad 0 \leq K \leq 0.8 \ ;$$

$$S^{(\ell)}(K) = a_3^{(\ell)} K^2 + a_4^{(\ell)} K + a_5^{(\ell)} + a_6^{(\ell)} K^{-1} + a_7^{(\ell)} K^{-2}, \qquad 0.8 \leq K \leq 2 \ ;$$

$$\left. \begin{array}{l} S^{(1)}(K) = a_8^{(1)} K + a_9^{(1)} + a_{10}^{(1)} K^{-1} + a_{11}^{(1)} K^{-2} \\[2mm] S^{(\ell)}(K) = a_8^{(\ell)} + a_9^{(\ell)} K^{-1} + a_{10}^{(\ell)} K^{-2} + a_{11}^{(\ell)} K^{-3}, \ \ell \neq 1 \end{array} \right\} \qquad 2 \leq K \leq 10 \ ; \tag{23}$$

$$S^{(\ell)}(K) = a_{12}^{(\ell)} + a_{13}^{(\ell)} K^{-\frac{1}{2}} + a_{14}^{(\ell)} K^{-1} + a_{15}^{(\ell)} K^{-\frac{3}{2}}, \qquad 10 \leq K < \infty \ .$$

Table III. Cross section functions, $R^{(\ell)}(K)$.

$$R^{(1)}(K)$$

K \ α	12	13	14	15
0	1.5778	1.5778	1.5778	1.5778
0.4	1.5701	1.5588	1.5495	1.5490
0.78101	1.5327			
0.8		1.5280	1.5277	1.5275
1.0	1.5178	1.5297	1.5320	1.5282
1.2	1.4083	1.4392	1.4566	1.4811
1.6	1.2271	1.2714	1.3027	1.3282
2	1.1336	1.1743	1.2073	1.2350
3	1.0361	1.0805	1.1177	1.1509
4	1.0119	1.0613	1.1018	1.1366
5	1.0109	1.0626	1.1070	1.1443
10	1.0646	1.1297	1.1865	1.2356
20	1.1546	1.2401	1.3107	1.3769
50	1.2824	1.4032	1.5069	1.6000
100	1.3676	1.5226	1.6579	1.7811

$$R^{(2)}(K)$$

K \ α	12	13	14	15
0	1.7865	1.7865	1.7865	1.7865
0.4	1.7923	1.7936	1.7985	1.7980
0.78101	1.7991			
0.8		1.8001	1.8060	1.8045
1.0	1.8357	1.7864	1.7959	1.7984
1.2	1.8941	1.8727	1.8627	1.8304
1.6	1.8344	1.8688	1.8826	1.3918
2	1.6929	1.7333	1.7587	1.7726
3	1.4442	1.4921	1.5288	1.5621
4	1.3418	1.3870	1.4305	1.4619
5	1.2986	1.3489	1.3918	1.4274
10	1.3051	1.3676	1.4200	1.4689
20	1.4141	1.4972	1.5605	1.6272
50	1.6063	1.7233	1.8294	1.9171
100	1.7599	1.9129	2.0461	2.1613

sections are independent of the potential energy of repulsion, and depend on the attraction energy alone. From this it is easy to show that the $S^{(\ell)}(K)$ are proportional to $\left[K(1 - 6/\alpha)\right]^{-1/3}$ for $K \to 0$; we therefore define the functions $R^{(\ell)}(K)$:

$$R^{(\ell)}(K) = \left[K(1 - 6/\alpha)\right]^{1/3} S^{(\ell)}(K),\tag{22}$$

which are given in Table III, and plotted for $\alpha = 12$ and 15 in Figure 2 for the two most important cases, $\ell = 1$ and 2. For comparison, the analogous functions for the Lennard-Jones (12-6) potential are also plotted. Since $R^{(\ell)}(0)$ is independent of the repulsion, the values given in the table for $\ell = 1$ and 2 were taken directly from Kihara and Kotani[5]. For $\ell = 3$ and 4, the values of $R^{(\ell)}(0)$ were obtained by extrapolation, a procedure which is permissible because these functions occur in the expressions for the transport properties only as correction terms.

For use in evaluating the collision integrals, the cross section functions were approximated by the following algebraic expressions:

$$R^{(\ell)}(K) = R^{(\ell)}(0)\left(1 + a_1^{(\ell)} K^{\frac{1}{3}} + a_2^{(\ell)} K^{\frac{2}{3}}\right), \qquad 0 \le K \le 0.8\,;$$

$$S^{(\ell)}(K) = a_3^{(\ell)} K^2 + a_4^{(\ell)} K + a_5^{(\ell)} + a_6^{(\ell)} K^{-1} + a_7^{(\ell)} K^{-2}, \qquad 0.8 \le K \le 2\,;$$

$$\left.\begin{array}{l} S^{(1)}(K) = a_8^{(1)} K + a_9^{(1)} + a_{10}^{(1)} K^{-1} + a_{11}^{(1)} K^{-2} \\[2mm] S^{(\ell)}(K) = a_8^{(\ell)} + a_9^{(\ell)} K^{-1} + a_{10}^{(\ell)} K^{-2} + a_{11}^{(\ell)} K^{-3}, \ell \neq 1 \end{array}\right\} \qquad 2 \le K \le 10\,; \tag{23}$$

$$S^{(\ell)}(K) = a_{12}^{(\ell)} + a_{13}^{(\ell)} K^{-\frac{1}{2}} + a_{14}^{(\ell)} K^{-1} + a_{15}^{(\ell)} K^{-\frac{3}{2}}, \qquad 10 \le K < \infty\,.$$

Table III. Cross section functions, $R^{(\ell)}(K)$. Continued

$R^{(3)}(K)$

K \ α	12	13	14	15
0	(1.8236)	(1.8236)	(1.8236)	(1.8236)
0.4	1.8115	1.8005	1.7938	1.7987
0.78101	1.7831			
0.8		1.7749	1.7821	1.7745
1.0	1.7796	1.7833	1.7778	1.7797
1.2	1.6788	1.6930	1.7042	1.7285
1.6	1.6121	1.6333	1.6433	1.6595
2	1.5325	1.5665	1.5884	1.6151
3	1.3587	1.4012	1.4376	1.4692
4	1.2689	1.3167	1.3557	1.3934
5	1.2256	1.2776	1.3206	1.3574
10	1.2222	1.2875	1.3447	1.3893
20	1.3151	1.3993	1.4632	1.5363
50	1.4922	1.6047	1.7045	1.7928
100	1.6115	1.7641	1.8980	2.0141

$R^{(4)}(K)$

K \ α	12	13	14	15
0	(1.9100)	(1.9100)	(1.9100)	(1.9100)
0.4	1.9335	1.9208	1.9225	1.9274
0.78101	1.9345			
0.8		1.9178	1.9221	1.9586
1.0	1.9330	1.9191	1.9065	1.9107
1.2	2.0278	1.9948	1.9907	1.9674
1.6	1.9600	1.9878	1.9770	1.9983
2	1.8895	1.9176	1.9300	1.9258
3	1.6584	1.7026	1.7363	1.7663
4	1.5197	1.5623	1.6045	1.6348
5	1.4433	1.4941	1.5328	1.5705
10	1.3868	1.4514	1.4987	1.5488
20	1.4838	1.5670	1.6252	1.6893
50	1.6931	1.8089	1.9105	1.9954
100	1.8636	2.0136	2.1450	2.2558
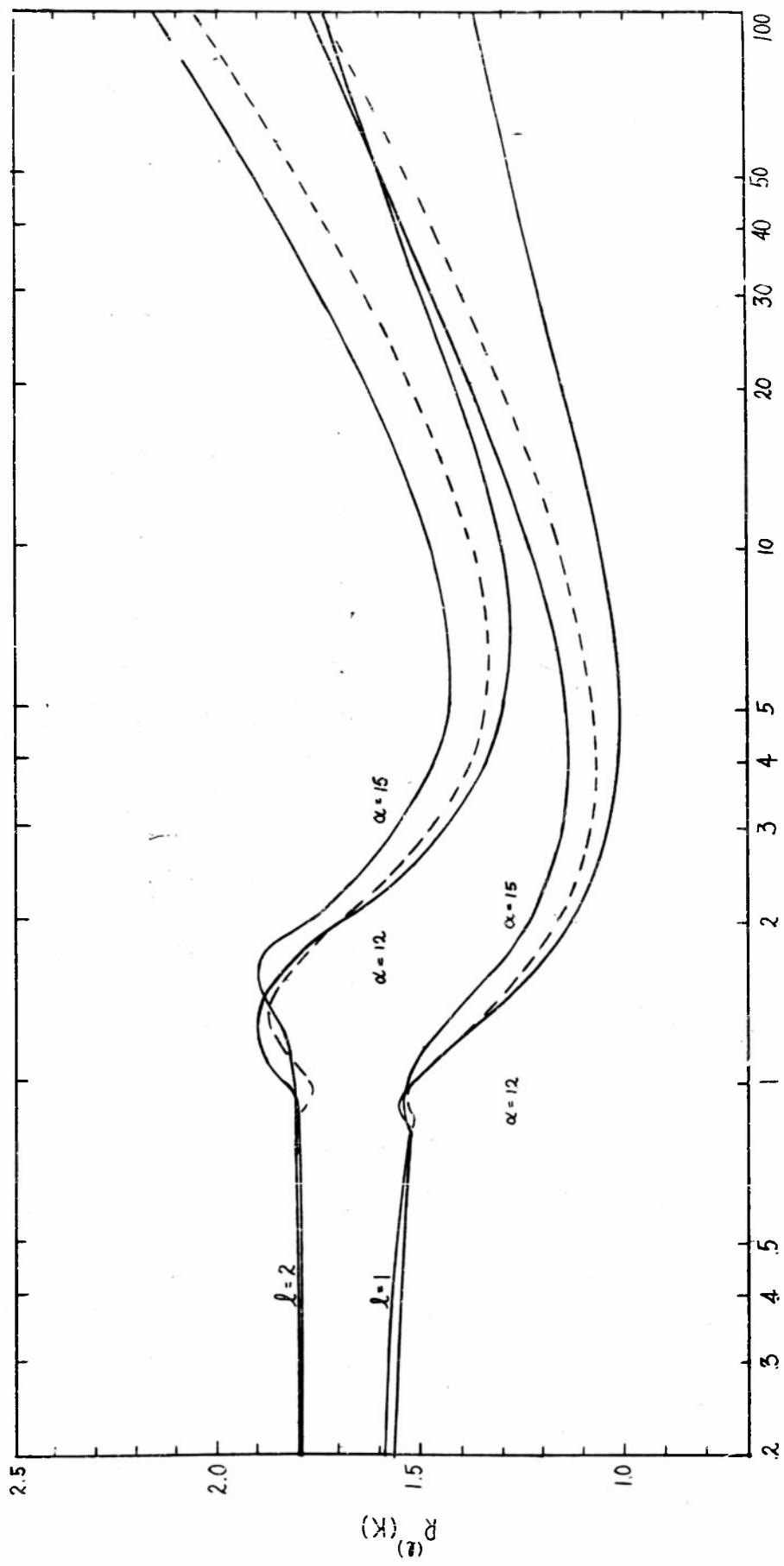
Fig. 2. The cross section functions, $R^{(\ell)}(K)$. Solid lines: Buckingham potential; broken lines: Lennard-Jones (12-6) potential.

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This curve-fitting procedure effectively permits the extrapolation of the $S^{(\ell)}(K)$ to infinite K, which is necessary because the potential energy function $\varphi(r)$ of Eq. (1) misbehaves for very small values of r by becoming negative.

The final integrations with respect to K to obtain the collision integrals defined in Eq. (5) could be carried out by replacing $S^{(\ell)}(K)$ in Eq. (5) by the algebraic expressions obtained from Eqs. (22) and (23). For $K > 0.8$, the resulting integrals could be easily evaluated in terms of elementary functions and error integrals, which are tabulated functions. For $K < 0.8$, the integrations were performed numerically for large T^*, where this region did not contribute greatly to the result, and for small T^* by means of incomplete gamma functions tabulated by Kotani[33]. In practice it turned out to be easier also to integrate numerically for $K > 0.8$ when T^* was large. For numerical integrations, the algebraic expressions in Eq. (23) were used only as interpolation functions to obtain values of $S^{(\ell)}(K)$ at small intervals of K.

The collision integrals are not especially well suited to tabulation because their rapid variation for small T^* makes interpolation difficult unless the tabulations are made for small intervals of T^*. A more suitable function for tabulation is

$$Z^{(\ell, n)}(T^*) = \left[T^*(1 - 6/\alpha) \right]^{1/3} \Omega^{(\ell, n)*}(T^*) . \qquad (24)$$

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[33] M. Kotani, Proc. Phys.-Math. Soc. Japan <u>24</u>, 76 (1942).

This function varies relatively slowly with $T*$, and unlike $\Omega^{(\ell, n)*}(T*)$ is finite and independent of the parameter α when $T* = 0$. The functions $Z^{(\ell, n)}(T*)$ are tabulated in Table IV for $T*$ from 0 to 200. The inherent computational error is estimated to be of the order of 0.2 percent, except for very small $T*$, where it may be as large as one percent. To maintain internal consistency, more figures are given in Table IV than is strictly justified by the estimated error. The reason for the lower estimated accuracy at small $T*$ is that the cross section functions $R^{(\ell)}(K)$ seem to have an oscillatory behavior when K is small, but no attempt was made to follow this behavior in detail since many calculations at closely spaced values of K would have been necessary. The labor involved did not seem worthwhile, particularly since the neglect of quantum corrections at low K is probably a much more serious error.

IV. ADDITIONAL TABULATIONS

In Table V are presented the functions, f, which give the higher approximations to the transport coefficients for pure gases; the superscripts indicate that these are the third approximations rather than the exact values. The function f_D for a pure gas refers to the coefficient of self-diffusion.

The second approximation to the reduced thermal diffusion ratio for a binary mixture of heavy isotopes, $\left[k_T^*\right]_2$, is given in Table VI as a function of $T*$, and plotted in Figure 3 for $\alpha = 12$ and 15. For comparison, the function $\left[k_T^*\right]_1$ for the Lennard-Jones (12-6) potential, as calculated by the method of Chapman and Cowling, is also shown in Figure 3. It is of interest to note that $\left[k_T^*\right]_2$ is always positive for $\alpha = 15$, showing that there is no

inversion temperature when the repulsive portion of the potential φ (r) is steep enough.

Table VII gives the values of the ratios A* , B* , and C* which occur in the expressions for the transport properties of mixtures The dimensionless group $\rho D_{11} / \eta$, frequently encountered in kinetic theory, is given by $(6f_D / 5f_\eta)A^*$.

Experimental viscosity results are often expressed by an equation of the form

$$\eta = cT^{s_\eta} ,$$ (25)

where c and s_η are supposed to be constants for a particular gas. The temperature index, s_η , is seen to be given by

$$s_\eta = d \log \eta / d \log T.$$ (26)

The temperature index has the following physical significance: if the intermolecular potential were of the form φ (r) = const. $/r^\nu$, then s_η would be constant and equal to $\left[1/2 + 2/(\nu - 1) \right]$. We can investigate the behavior of s_η for the potential given by Eq. (1) by requiring that the values of η and $d\eta$ /dT determined from Eq. (25) agree with those from Eq. (6) at a given temperature. In this way we find that s_η is not a constant, but the following function of T* :

$$s_\eta = \frac{9}{2} - \frac{4 \Omega^{(2,3)^*}}{\Omega^{(2,2)^*}} + \frac{d \log f_\eta}{d \log T^*} ,$$ (27)

where we have made use of the recursion relation

$$\frac{d \log \Omega^{(\ell,n)^*}}{d \log T^*} = (n+2)\left(\frac{\Omega^{(\ell,n+1)^*}}{\Omega^{(\ell,n)*}} - 1\right) .$$ (28)

Another common method of expressing experimental viscosity results is by the Sutherland equation,

$$\eta = \frac{c' T^{\frac{1}{2}}}{1 + S_\eta/T} ,$$ (29)

where c' and S_η are supposed to be constants for a particular gas. This equation is based on the physical model of rigid spheres which have a weak attraction for one another. The physical significance of the Sutherland constant, S_η, is that it is proportional to the potential energy of two such spheres when in contact. We can find the dependence of S_η on T^* in the same way that the behavior of s_η was found; it is

$$\frac{S_n}{T} = \left[4\left(1 - \frac{\Omega^{(2,3)^*}}{\Omega^{(2,2)*}}\right) + \frac{d \log f_n}{d \log T^*}\right] \bigg/ \left[1 - 4\left(1 - \frac{\Omega^{(2,3)^*}}{\Omega^{(2,2)*}}\right) + \frac{d \log f_\eta}{d \log T^*}\right] .$$ (30)

An entirely analogous result holds for the coefficient of thermal conductivity, λ. The expressions for s_λ and S_λ/T are the same as given in Eqs. (27) and (30), respectively, except that f_η is replaced by f_λ.

In a similar way, we can define a temperature index for self-diffusion,

$$s_D = d \log D_{11}/d \log T ,$$ (31)

and a Sutherland constant for self-diffusion.

$$D_{11} = \frac{c'' \, T^{\frac{1}{2}}}{1 + S_D/T} \qquad (32)$$

If D_{11} in Eq. (31) is expressed at <u>constant density</u>, we find that the temperature index s_D is the following function of T^* :

$$s_D = \frac{7}{2} - \frac{3\,\Omega^{(1,2)^*}}{\Omega^{(1,1)*}} + \frac{d \log f_D}{d \log T^*} \qquad \qquad (33)$$

For the Sutherland constant S_D/T we obtain

$$\frac{S_D}{T} = \left[3\left(1 - \frac{\Omega^{(1,2)*}}{\Omega^{(1,1)*}} \right) + \frac{d \log f_D}{d \log T^*} \right] \Big/ \left[1 - 3\left(1 - \frac{\Omega^{(1,2)*}}{\Omega^{(1,1)*}} \right) + \frac{d \log f_D}{d \log T^*} \right] , \qquad (34)$$

independent of whether D_{11} is expressed at constant density or constant pressure.

Values of the temperature indexes for viscosity, thermal conductivity, and self-diffusion were calculated from Eqs. (27) and (33) neglecting the terms involving f , and are given as a function of T^* in Table VIIIA. The error introduced by neglecting the terms in f is in all cases less than one percent. In Figure 4A is plotted s_η vs. T^* for $\alpha = 12$ and 15 , together with the corresponding s_η for the Lennard-Jones (12-6) potential for comparison. Values of the Sutherland constants are given in Table VIIIB, and plotted in Figure 4B for $\alpha = 12$ and 15 , and the Lennard-Jones (12-6) potential. The neglect of the terms involving f in the calculations of the Sutherland constants introduced an error of less than one percent.

Numerical values of the potential parameters of Eq. (1) for a particular gas may be calculated by comparison of the experimentally determined values of S/T or s with the theoretical values in Table VIII. Caution must be exercised if this procedure is used, however, because inconsistent results may be obtained unless the experimental quantities are calculated in the same way as the theoretical quantities were; that is, from both the temperature derivative and the numerical value of the transport coefficient at a given temperature.

Table IV   Collision Integral Functions

$$Z^{(l,n)}(T^*) = [T^*(1-6/\alpha)]^{1/3} \Omega^{(l,n)*}(T^*) \qquad\qquad T^* = kT/\epsilon$$

$\alpha = 12$

T*	Z(1, 1)	Z(1, 2)	Z(1, 3)	Z(1, 4)	Z(1, 5)	Z(3, 3)
0	1.1870	1.0551	0.9672	0.9027	0.8572	1.1178
.1	1.1911	1.0523	0.9584	0.8883	0.8320	1.1079
.2	1.1662	1.0142	0.9025	0.8113	0.7338	1.0704
.3	1.1243	0.9515	0.8232	0.7239	0.6475	1.0161
.4	1.0750	0.8900	0.7597	0.6667	0.6005	0.9641
.5	1.0282	0.8402	0.7160	0.6328	0.5764	0.9194
.6	0.9873	0.8025	0.6870	0.6131	0.5643	0.8828
.7	0.9530	0.7745	0.6679	0.6019	0.5589	0.8536
.8	0.9248	0.7538	0.6555	0.5959	0.5572	0.8305
.9	0.9016	0.7387	0.6475	0.5930	0.5576	0.8125
1.0	0.8825	0.7272	0.6425	0.5923	0.5593	0.7985
1.2	0.8541	0.7130	0.6385	0.5942	0.5646	0.7793
1.4	0.8350	0.7058	0.6388	0.5985	0.5711	0.7684
1.6	0.8221	0.7028	0.6414	0.6039	0.5779	0.7625
1.8	0.8135	0.7025	0.6451	0.6096	0.5845	0.7601
2.0	0.8080	0.7037	0.6496	0.6155	0.5910	0.7600
2.5	0.8023	0.7104	0.6616	0.6296	0.6059	0.7654
3.0	0.8031	0.7195	0.6735	0.6425	0.6180	0.7746
3.5	0.8070	0.7288	0.6847	0.6541	0.6305	0.7853
4	0.8126	0.7382	0.6950	0.6644	0.6404	0.7963
5	0.8253	0.7556	0.7129	0.6819	0.6572	0.8174
6	0.8384	0.7709	0.7280	0.6962	0.6705	0.8364
7	0.8509	0.7842	0.7409	0.7081	0.6817	0.8533
8	0.8623	0.7961	0.7520	0.7185	0.6912	0.8680
9	0.8730	0.8067	0.7618	0.7272	0.6993	0.8813
10	0.8829	0.8160	0.7702	0.7351	0.7066	0.8929
12	0.9002	0.8322	0.7848	0.7485	0.7189	0.9133
14	0.9150	0.8457	0.7969	0.7594	0.7292	0.9301
16	0.9280	0.8572	0.8074	0.7690	0.7380	0.9444
18	0.9394	0.8674	0.8164	0.7773	0.7459	0.9568
20	0.9497	0.8762	0.8245	0.7846	0.7530	0.9680
25	0.9710	0.8951	0.8415	0.8007	0.7682	0.9910
30	0.9885	0.9103	0.8555	0.8138	0.7808	1.0094
35	1.0032	0.9232	0.8674	0.8253	0.7921	1.0250
40	1.0157	0.9346	0.8781	0.8355	0.8021	1.0385
45	1.0270	0.9446	0.8876	0.8450	0.8114	1.0508
50	1.0371	0.9538	0.8965	0.8535	0.8199	1.0614
60	1.0549	0.9701	0.9123	0.8691	0.8355	1.0807
70	1.0700	0.9846	0.9264	0.8827	0.8495	1.0978
80	1.0834	0.9973	0.9388	0.8957	0.8622	1.1129
90	1.0959	1.0091	0.9504	0.9074	0.8743	1.1265
100	1.1071	1.0201	0.9615	0.9184	0.8853	1.1391
200	1.1906	1.1038	1.0462	1.0049	0.9729	1.2356

Table IV   Collision Integral Functions continued

$\alpha = 12$

T	$Z^{(2,2)}$	$Z^{(2,3)}$	$Z^{(2,4)}$	$Z^{(2,5)}$	$Z^{(2,6)}$	$Z^{(4,4)}$
0	1.1947	1.0951	1.0221	0.9706	0.9193	1.0927
.1	1.1983	1.1002	1.0291	0.9750	0.9323	1.1062
.2	1.2098	1.1094	1.0401	0.9770	0.9188	1.1119
.3	1.2065	1.0936	0.9978	0.9129	0.8381	1.0843
	1.1799	1.0471	0.9367	0.8453	0.7711	1.0360
.5	1.1421	0.9974	0.8835	0.7953	0.7280	0.9870
.6	1.1028	0.9538	0.8428	0.7614	0.7017	0.9450
.7	1.0667	0.9185	0.8133	0.7391	0.6862	0.9111
.8	1.0352	0.8907	0.7923	0.7249	0.6775	0.8847
.9	1.0084	0.8692	0.7775	0.7160	0.6731	0.8642
1.0	0.9859	0.8528	0.7673	0.7108	0.6716	0.8488
1.2	0.9520	0.8309	0.7561	0.7075	0.6736	0.8282
1.4	0.9290	0.8189	0.7525	0.7093	0.6790	0.8170
1.6	0.9137	0.8130	0.7531	0.7139	0.6860	0.8120
1.8	0.9039	0.8111	0.7560	0.7198	0.6936	0.8107
2.0	0.8978	0.8116	0.7604	0.7263	0.7014	0.8119
2.5	0.8930	0.8189	0.7742	0.7435	0.7202	0.8207
3.0	0.8963	0.8300	0.7891	0.7600	0.7374	0.8331
3.5	0.9033	0.8423	0.8035	0.7752	0.7529	0.8466
4	0.9122	0.8546	0.8171	0.7892	0.7668	0.8599
5	0.9313	0.8778	0.8415	0.8135	0.7906	0.8849
6	0.9503	0.8987	0.8623	0.8339	0.8105	0.9072
7	0.9681	0.9172	0.8806	0.8516	0.8275	0.9269
8	0.9845	0.9339	0.8967	0.8670	0.8424	0.9445
9	0.9995	0.9488	0.9110	0.8808	0.8559	0.9602
10	1.0133	0.9624	0.9241	0.8935	0.8678	0.9745
12	1.0381	0.9862	0.9469	0.9153	0.8891	0.9998
14	1.0598	1.0068	0.9664	0.9341	0.9073	1.0213
16	1.0788	1.0248	0.9838	0.9508	0.9236	1.0404
18	1.0958	1.0409	0.9993	0.9658	0.9383	1.0575
20	1.1113	1.0557	1.0134	0.9796	0.9516	1.0731
25	1.1448	1.0875	1.0441	1.0095	0.9812	1.1070
30	1.1729	1.1145	1.0703	1.0353	1.0067	1.1357
35	1.1976	1.1382	1.0935	1.0582	1.0294	1.1608
40	1.2193	1.1593	1.1143	1.0787	1.0499	1.1832
45	1.2393	1.1786	1.1332	1.0976	1.0688	1.2041
50	1.2573	1.1962	1.1509	1.1152	1.0863	1.2228
60	1.2901	1.2283	1.1826	1.1469	1.1180	1.2572
70	1.3189	1.2571	1.2113	1.1756	1.1468	1.2878
80	1.3451	1.2828	1.2370	1.2014	1.1730	1.3157
90	1.3690	1.3068	1.2609	1.2257	1.1973	1.3413
100	1.3915	1.3292	1.2835	1.2481	1.2198	1.3649
200	1.5600	1.4988	1.4547	1.4208	1.3939	1.5470

Table IV. Collision Integral Functions continued

$\alpha = 13$

T*	$Z^{(1,\ 1)}$	$Z^{(1,\ 2)}$	$Z^{(1,\ 3)}$	$Z^{(1,\ 4)}$	$Z^{(1,\ 5)}$	$Z^{(3,\ 3)}$
0	1.1870	1.0551	0.9672	0.9027	0.8572	1.1178
.1	1.1813	1.0447	0.9528	0.8846	0.8305	1.1020
.2	1.1618	1.0150	0.9084	0.8217	0.7479	1.0718
.3	1.1275	0.9619	0.8389	0.7429	0.6681	1.0250
.4	1.0851	0.9071	0.7807	0.6893	0.6236	0.9787
.5	1.0436	0.8617	0.7400	0.6575	0.6011	0.9383
.6	1.0068	0.8268	0.7129	0.6393	0.5904	0.9048
.7	0.9756	0.8009	0.6953	0.6294	0.5862	0.8780
.8	0.9498	0.7818	0.6841	0.6244	0.5855	0.8569
.9	0.9285	0.7678	0.6772	0.6226	0.5871	0.8405
1.0	0.9112	0.7577	0.6732	0.6229	0.5897	0.8279
1.2	0.8854	0.7454	0.6710	0.6266	0.5970	0.8111
1.4	0.8684	0.7400	0.6729	0.6326	0.6052	0.8022
1.6	0.8573	0.7386	0.6770	0.6395	0.6136	0.7981
1.8	0.8503	0.7396	0.6823	0.6468	0.6219	0.7974
2.0	0.8461	0.7421	0.6881	0.6542	0.6299	0.7987
2.5	0.8436	0.7519	0.7032	0.6715	0.6483	0.8073
3.0	0.8472	0.7637	0.7181	0.6875	0.6643	0.8193
3.5	0.8535	0.7757	0.7320	0.7017	0.6785	0.8323
4	0.8614	0.7874	0.7446	0.7145	0.6910	0.8454
5	0.8782	0.8091	0.7671	0.7365	0.7121	0.8700
6	0.8950	0.8282	0.7862	0.7547	0.7296	0.8922
7	0.9107	0.8451	0.8024	0.7702	0.7441	0.9118
8	0.9253	0.8601	0.8166	0.7836	0.7568	0.9294
9	0.9389	0.8734	0.8292	0.7954	0.7678	0.9453
10	0.9514	0.8855	0.8404	0.8059	0.7777	0.9596
12	0.9736	0.9065	0.8599	0.8240	0.7948	0.9847
14	0.9927	0.9245	0.8765	0.8394	0.8094	1.0063
16	1.0096	0.9399	0.8907	0.8528	0.8223	1.0250
18	1.0247	0.9537	0.9034	0.8648	0.8337	1.0417
20	1.0384	0.9659	0.9147	0.8754	0.8440	1.0569
25	1.0674	0.9922	0.9392	0.8987	0.8664	1.0895
30	1.0913	1.0139	0.9596	0.9184	0.8855	1.1166
35	1.1118	1.0328	0.9775	0.9357	0.9027	1.1403
40	1.1296	1.0492	0.9933	0.9510	0.9179	1.1616
45	1.1456	1.0643	1.0076	0.9651	0.9318	1.1806
50	1.1602	1.0778	1.0208	0.9783	0.9447	1.1983
60	1.1861	1.1023	1.0447	1.0017	0.9682	1.2297
70	1.2087	1.1239	1.0659	1.0230	0.9894	1.2577
80	1.2287	1.1431	1.0849	1.0422	1.0089	1.2830
90	1.2469	1.1608	1.1025	1.0598	1.0267	1.3063
100	1.2639	1.1774	1.1193	1.0762	1.0434	1.3277
200	1.3892	1.3027	1.2456	1.2042	1.1723	1.4915

Table IV.   Collision Integral Functions continued

α = 13

T*	z(2, 2)	z(2, 3)	z(2, 4)	z(2, 5)	z(2, 6)	z(4, 4)
0	1.1947	1.0951	1.0221	0.9706	0.9193	1.0927
.1	1.1985	1.0994	1.0269	0.9709	0.9265	1.0986
.2	1.2052	1.1090	1.0376	0.9783	0.9247	1.1116
.3	1.2056	1.0977	1.0077	0.9278	0.8562	1.0944
.4	1.1862	1.0602	0.9549	0.8664	0.7934	1.0526
.5	1.1549	1.0164	0.9059	0.8190	0.7519	1.0076
.6	1.1207	0.9766	0.8676	0.7866	0.7267	0.9681
.7	1.0882	0.9436	0.8395	0.7653	0.7120	0.9360
.8	1.0594	0.9175	0.8196	0.7519	0.7043	0.9110
.9	1.0346	0.8972	0.8057	0.7439	0.7009	0.8917
1.0	1.0138	0.8820	0.7965	0.7398	0.7005	0.8775
1.2	0.9824	0.8618	0.7870	0.7382	0.7044	0.8590
1.4	0.9613	0.8514	0.7850	0.7418	0.7118	0.8499
1.6	0.9475	0.8471	0.7871	0.7481	0.7206	0.8466
1.8	0.9391	0.8465	0.7916	0.7557	0.7299	0.8469
2.0	0.9344	0.8484	0.7976	0.7638	0.7392	0.8496
2.5	0.9327	0.8590	0.8148	0.7844	0.7615	0.8617
3.0	0.9388	0.8731	0.8326	0.8040	0.7817	0.8771
3.5	0.9485	0.8880	0.8497	0.8219	0.7998	0.8931
4	0.9597	0.9027	0.8658	0.8381	0.8161	0.9087
5	0.9831	0.9303	0.8944	0.8668	0.8443	0.9378
6	1.0058	0.9548	0.9191	0.8911	0.8679	0.9637
7	1.0270	0.9769	0.9408	0.9121	0.8885	0.9867
8	1.0464	0.9964	0.9600	0.9309	0.9066	1.0072
9	1.0643	1.0143	0.9773	0.9477	0.9230	1.0257
10	1.0809	1.0306	0.9929	0.9628	0.9377	1.0425
12	1.1105	1.0592	1.0207	0.9898	0.9641	1.0725
14	1.1363	1.0843	1.0449	1.0133	0.9871	1.0984
16	1.1595	1.1064	1.0662	1.0342	1.0076	1.1216
18	1.1801	1.1264	1.0857	1.0530	1.0262	1.1422
20	1.1991	1.1448	1.1033	1.0704	1.0432	1.1611
25	1.2406	1.1847	1.1423	1.1085	1.0812	1.2027
30	1.2758	1.2187	1.1757	1.1416	1.1138	1.2382
35	1.3066	1.2486	1.2052	1.1709	1.1430	1.2694
40	1.3344	1.2757	1.2320	1.1975	1.1694	1.2974
45	1.3595	1.3004	1.2565	1.2217	1.1937	1.3230
50	1.3826	1.3232	1.2792	1.2444	1.2165	1.3466
60	1.4243	1.3644	1.3202	1.2854	1.2577	1.3896
70	1.4615	1.4015	1.3569	1.3224	1.2945	1.4280
80	1.4951	1.4348	1.3906	1.3559	1.3282	1.4628
90	1.5262	1.4656	1.4215	1.3869	1.3595	1.4948
100	1.5546	1.4942	1.4500	1.4161	1.3885	1.5248
200	1.7718	1.7128	1.6699	1.6376	1.6114	1.7518

Table IV.   Collision Integral Functions <u>continued</u>

$\alpha = 14$

T*	$z^{(1, 1)}$	$z^{(1, 2)}$	$z^{(1, 3)}$	$z^{(1, 4)}$	$z^{(1, 5)}$	$z^{(3, 3)}$
0	1.1870	1.0551	0.9672	0.9027	0.8572	1.1178
.1	1.1727	1.0382	0.9481	0.8814	0.8287	1.0987
.2	1.1575	1.0146	0.9115	0.8281	0.7571	1.0729
.3	1.1286	0.9684	0.8497	0.7566	0.6836	1.0308
.4	1.0914	0.9191	0.7963	0.7069	0.6420	0.9891
.5	1.0542	0.8776	0.7585	0.6773	0.6214	0.9524
.6	1.0208	0.8455	0.7336	0.6608	0.6121	0.9218
.7	0.9924	0.8217	0.7176	0.6521	0.6091	0.8974
.8	0.9688	0.8042	0.7076	0.6483	0.6096	0.8781
.9	0.9495	0.7917	0.7018	0.6476	0.6121	0.8633
1.0	0.9337	0.7827	0.6989	0.6488	0.6157	0.8519
1.2	0.9106	0.7724	0.6984	0.6542	0.6246	0.8373
1.4	0.8956	0.7686	0.7019	0.6617	0.6342	0.8300
1.6	0.8863	0.7687	0.7074	0.6700	0.6440	0.8273
1.8	0.8808	0.7710	0.7139	0.6784	0.6534	0.8277
2.0	0.8780	0.7747	0.7208	0.6868	0.6625	0.8300
2.5	0.8784	0.7871	0.7385	0.7068	0.6836	0.8406
3.0	0.8843	0.8011	0.7557	0.7252	0.7022	0.8545
3.5	0.8928	0.8152	0.7716	0.7416	0.7188	0.8692
4	0.9025	0.8288	0.7863	0.7565	0.7335	0.8840
5	0.9228	0.8539	0.8125	0.7824	0.7587	0.9121
6	0.9426	0.8764	0.8349	0.8042	0.7796	0.9376
7	0.9612	0.8962	0.8543	0.8229	0.7974	0.9605
8	0.9784	0.9140	0.8715	0.8391	0.8129	0.9812
9	0.9944	0.9300	0.8867	0.8536	0.8266	0.9999
10	1.0092	0.9445	0.9005	0.8666	0.8390	1.0169
12	1.0358	0.9701	0.9245	0.8893	0.8606	1.0468
14	1.0588	0.9918	0.9450	0.9086	0.8792	1.0726
16	1.0794	1.0110	0.9629	0.9255	0.8956	1.0953
18	1.0978	1.0280	0.9789	0.9408	0.9101	1.1157
20	1.1143	1.0436	0.9931	0.9546	0.9235	1.1339
25	1.1501	1.0766	1.0244	0.9844	0.9526	1.1737
30	1.1802	1.1044	1.0507	1.0097	0.9772	1.2070
35	1.2060	1.1281	1.0736	1.0320	0.9992	1.2359
40	1.2286	1.1494	1.0940	1.0520	1.0191	1.2615
45	1.2491	1.1685	1.1125	1.0703	1.0372	1.2848
50	1.2678	1.1862	1.1296	1.0871	1.0538	1.3063
60	1.3008	1.2176	1.1604	1.1179	1.0844	1.3446
70	1.3297	1.2455	1.1877	1.1450	1.1115	1.3789
80	1.3555	1.2708	1.2125	1.1699	1.1363	1.4095
90	1.3789	1.2938	1.2354	1.1926	1.1595	1.4377
100	1.4009	1.3150	1.2568	1.2141	1.1809	1.4637
200	1.5626	1.4758	1.4185	1.3768	1.3452	1.6611

Table IV.   Collision Integral Functions continued

$\alpha = 14$

T*	$Z^{(2, 2)}$	$Z^{(2, 3)}$	$Z^{(2, 4)}$	$Z^{(2, 5)}$	$Z^{(2, 6)}$	$Z^{(4, 4)}$
0	1.1947	1.0951	1.0221	0.9706	0.9193	1.0927
.1	1.2017	1.1027	1.0301	0.9742	0.9296	1.0990
.2	1.2090	1.1128	1.0416	0.9831	0.9309	1.1098
.3	1.2109	1.1046	1.0168	0.9391	0.8695	1.0965
.4	1.1947	1.0716	0.9690	0.8825	0.8109	1.0607
.5	1.1670	1.0317	0.9235	0.8382	0.7718	1.0206
.6	1.1359	0.9948	0.8878	0.8076	0.7481	0.9846
.7	1.1060	0.9641	0.8615	0.7877	0.7346	0.9549
.8	1.0794	0.9397	0.8428	0.7754	0.7277	0.9315
.9	1.0564	0.9209	0.8300	0.7682	0.7249	0.9135
1.0	1.0370	0.9066	0.8215	0.7647	0.7250	0.9001
1.2	1.0078	0.8882	0.8132	0.7641	0.7299	0.8829
1.4	0.9885	0.8790	0.8122	0.7686	0.7380	0.8746
1.6	0.9761	0.8756	0.8151	0.7756	0.7476	0.8720
1.8	0.9688	0.8759	0.8205	0.7840	0.7580	0.8731
2.0	0.9649	0.8785	0.8271	0.7930	0.7683	0.8764
2.5	0.9651	0.8910	0.8464	0.8162	0.7937	0.8905
3.0	0.9729	0.9069	0.8666	0.8385	0.8172	0.9081
3.5	0.9844	0.9239	0.8862	0.8594	0.8384	0.9267
4	0.9973	0.9409	0.9048	0.8785	0.8575	0.9449
5	1.0244	0.9729	0.9384	0.9121	0.8908	0.9790
6	1.0507	1.0015	0.9675	0.9408	0.9188	1.0092
7	1.0755	1.0274	0.9882	0.9656	0.9428	1.0361
8	1.0982	1.0504	1.0154	0.9875	0.9639	1.0600
9	1.1192	1.0712	1.0357	1.0068	0.9827	1.0816
10	1.1385	1.0902	1.0539	1.0244	0.9996	1.1013
12	1.1731	1.1237	1.0859	1.0554	1.0293	1.1359
14	1.2034	1.1526	1.1136	1.0818	1.0550	1.1656
16	1.2302	1.1781	1.1379	1.1051	1.0777	1.1917
18	1.2540	1.2007	1.1596	1.1261	1.0980	1.2153
20	1.2758	1.2213	1.1792	1.1452	1.1168	1.2364
25	1.3229	1.2659	1.2222	1.1868	1.1576	1.2826
30	1.3622	1.3034	1.2583	1.2222	1.1925	1.3217
35	1.3963	1.3363	1.2902	1.2535	1.2231	1.3559
40	1.4266	1.3653	1.3185	1.2816	1.2510	1.3863
45	1.4543	1.3917	1.3445	1.3070	1.2766	1.4141
50	1.4793	1.4161	1.3684	1.3308	1.2999	1.4396
60	1.5239	1.4596	1.4115	1.3735	1.3427	1.4856
70	1.5636	1.4983	1.4497	1.4118	1.3806	1.5263
80	1.5990	1.5332	1.4846	1.4463	1.4156	1.5633
90	1.6314	1.5656	1.5165	1.4782	1.4474	1.5972
100	1.6612	1.5950	1.5461	1.5079	1.4771	1.6289
200	1.8858	1.8198	1.7713	1.7344	1.7043	1.8674

Table IV.   Collision Integral Functions continued

$\alpha = 15$

T*	$z^{(1, 1)}$	$z^{(1, 2)}$	$z^{(1, 3)}$	$z^{(1, 4)}$	$z^{(1, 5)}$	$z^{(3, 3)}$
0	1.1870	1.0551	0.9672	0.9027	0.8572	1.1178
.1	1.1722	1.0378	0.9477	0.8810	0.8285	1.1007
.2	1.1577	1.0163	0.9153	0.8343	0.7655	1.0765
.3	1.1320	0.9750	0.8594	0.7687	0.6970	1.0392
.4	1.0984	0.9300	0.8099	0.7221	0.6579	1.0012
.5	1.0645	0.8916	0.7747	0.6944	0.6388	0.9672
.6	1.0337	0.8618	0.7514	0.6792	0.6306	0.9387
.7	1.0075	0.8397	0.7367	0.6715	0.6285	0.9158
.8	0.9857	0.8235	0.7278	0.6687	0.6299	0.8978
.9	0.9678	0.8121	0.7230	0.6687	0.6333	0.8840
1.0	0.9533	0.8040	0.7208	0.6707	0.6378	0.8735
1.2	0.9323	0.7954	0.7218	0.6777	0.6484	0.8605
1.4	0.9191	0.7931	0.7266	0.6867	0.6596	0.8548
1.6	0.9112	0.7944	0.7335	0.6964	0.6708	0.8535
1.8	0.9070	0.7979	0.7412	0.7063	0.6817	0.8554
2.0	0.9054	0.8028	0.7494	0.7159	0.6921	0.8593
2.5	0.9083	0.8179	0.7699	0.7389	0.7160	0.8734
3.0	0.9167	0.8344	0.7896	0.7597	0.7372	0.8903
3.5	0.9273	0.8507	0.8078	0.7783	0.7558	0.9077
4	0.9389	0.8662	0.8245	0.7953	0.7725	0.9245
5	0.9627	0.8951	0.8541	0.8245	0.8012	0.9558
6	0.9856	0.9203	0.8796	0.8494	0.8250	0.9836
7	1.0069	0.9430	0.9017	0.8708	0.8458	1.0086
8	1.0268	0.9632	0.9214	0.8895	0.8638	1.0308
9	1.0449	0.9816	0.9390	0.9063	0.8800	1.0514
10	1.0619	0.9981	0.9549	0.9216	0.8946	1.0699
12	1.0922	1.0277	0.9829	0.9483	0.9205	1.1032
14	1.1188	1.0530	1.0070	0.9715	0.9428	1.1321
16	1.1426	1.0756	1.0284	0.9921	0.9628	1.1579
18	1.1640	1.0957	1.0475	1.0104	0.9805	1.1812
20	1.1834	1.1140	1.0648	1.0273	0.9970	1.2026
25	1.2257	1.1537	1.1029	1.0642	1.0333	1.2494
30	1.2614	1.1872	1.1353	1.0957	1.0642	1.2897
35	1.2923	1.2167	1.1637	1.1234	1.0920	1.3254
40	1.3199	1.2429	1.1890	1.1486	1.1169	1.3574
45	1.3446	1.2666	1.2123	1.1718	1.1397	1.3869
50	1.3675	1.2886	1.2339	1.1929	1.1609	1.4141
60	1.4083	1.3280	1.2726	1.2316	1.1996	1.4631
70	1.4443	1.3630	1.3073	1.2660	1.2340	1.5072
80	1.4766	1.3945	1.3389	1.2974	1.2654	1.5471
90	1.5059	1.4235	1.3675	1.3267	1.2946	1.5837
100	1.5331	1.4505	1.3945	1.3534	1.3217	1.6180
200	1.7372	1.6533	1.5991	1.5596	1.5291	1.8773

Table IV.   Collision Integral Functions continued

$\alpha = 15$

T*	$z^{(2,\ 2)}$	$z^{(2,\ 3)}$	$z^{(2,\ 4)}$	$z^{(2,\ 5)}$	$z^{(2,\ 6)}$	$z^{(4,\ 4)}$
0	1.1947	1.0951	1.0221	0.9706	0.9193	1.0927
.1	1.2016	1.1025	1.0306	0.9736	0.9290	1.1061
.2	1.2081	1.1114	1.0403	0.9828	0.9325	1.1150
.3	1.2107	1.1061	1.0211	0.9465	0.8796	1.1029
.4	1.1978	1.0783	0.9791	0.8952	0.8252	1.0705
.5	1.1739	1.0426	0.9374	0.8536	0.7879	1.0333
.6	1.1462	1.0088	0.9039	0.8247	0.7654	0.9996
.7	1.1191	0.9803	0.8791	0.8060	0.7529	0.9718
.8	1.0945	0.9575	0.8617	0.7946	0.7470	0.9499
.9	1.0734	0.9400	0.8498	0.7883	0.7454	0.9331
1.0	1.0555	0.9268	0.8423	0.7858	0.7466	0.9209
1.2	1.0285	0.9102	0.8359	0.7872	0.7535	0.9057
1.4	1.0110	0.9027	0.8365	0.7936	0.7637	0.8992
1.6	1.0003	0.9009	0.8412	0.8024	0.7751	0.8982
1.8	0.9944	0.9027	0.8481	0.8124	0.7868	0.9007
2.0	0.9920	0.9067	0.8562	0.8227	0.7986	0.9054
2.5	0.9953	0.9223	0.8786	0.8488	0.8266	0.9222
3.0	1.0059	0.9409	0.9013	0.8734	0.8521	0.9419
3.5	1.0196	0.9602	0.9229	0.8960	0.8750	0.9622
4	1.0345	0.9790	0.9431	0.9169	0.8958	0.9819
5	1.0651	1.0139	0.9796	0.9533	0.9318	1.0187
6	1.0943	1.0452	1.0111	0.9844	0.9623	1.0514
7	1.1213	1.0733	1.0389	1.0115	0.9887	1.0805
8	1.1462	1.0985	1.0634	1.0356	1.0121	1.1064
9	1.1691	1.1212	1.0856	1.0570	1.0332	1.1300
10	1.1904	1.1421	1.1059	1.0766	1.0523	1.1515
12	1.2283	1.1791	1.1416	1.1115	1.0860	1.1895
14	1.2616	1.2111	1.1727	1.1416	1.1154	1.2225
16	1.2911	1.2397	1.2002	1.1683	1.1417	1.2518
18	1.3179	1.2655	1.2250	1.1925	1.1656	1.2781
20	1.3423	1.2887	1.2477	1.2148	1.1873	1.3020
25	1.3954	1.3399	1.2975	1.2634	1.2353	1.3544
30	1.4406	1.3835	1.3400	1.3054	1.2768	1.3992
35	1.4802	1.4219	1.3775	1.3425	1.3138	1.4382
40	1.5152	1.4561	1.4114	1.3759	1.3471	1.4737
45	1.5474	1.4874	1.4424	1.4067	1.3776	1.5057
50	1.5769	1.5163	1.4710	1.4352	1.4060	1.5353
60	1.6298	1.5684	1.5225	1.4865	1.4575	1.5889
70	1.6768	1.6150	1.5687	1.5326	1.5034	1.6365
80	1.7190	1.6569	1.6107	1.5747	1.5453	1.6797
90	1.7580	1.6952	1.6491	1.6132	1.5841	1.7194
100	1.7938	1.7312	1.6850	1.6489	1.6200	1.7562
200	2.0647	2.0026	1.9572	1.9227	1.8945	2.0351

Table V. Functions for calculating the higher approximations to the transport coefficients.

The function $f_\eta^{(3)}$				
T*	$\alpha = 12$	$\alpha = 13$	$\alpha = 14$	$\alpha = 15$
0	1.0017	1.0017	1.0017	1.0017
.1	1.0019	1.0018	1.0018	1.0018
.2	1.0017	1.0021	1.0021	1.0020
.3	1.0012	1.0014	1.0015	1.0016
.4	1.0003	1.0005	1.0006	1.0008
.5	1.0001	1.0001	1.0002	1.0003
.6	1.0001	1.0000	1.0000	1.0001
.7	1.0002	1.0001	1.0000	1.0000
.8	1.0002	1.0001	1.0000	1.0000
.9	1.0002	1.0001	1.0000	1.0000
1.0	1.0001	1.0001	1.0000	1.0000
1.2	1.0001	1.0001	1.0001	1.0001
1.4	1.0001	1.0002	1.0002	1.0003
1.6	1.0003	1.0004	1.0005	1.0007
1.8	1.0005	1.0007	1.0008	1.0010
2.0	1.0008	1.0011	1.0012	1.0015
2.5	1.0017	1.0020	1.0022	1.0026
3.0	1.0025	1.0029	1.0031	1.0035
3.5	1.0032	1.0036	1.0039	1.0043
4	1.0037	1.0042	1.0046	1.0050
5	1.0045	1.0050	1.0055	1.0058
6	1.0050	1.0055	1.0061	1.0064
7	1.0052	1.0058	1.0066	1.0067
8	1.0054	1.0060	1.0067	1.0070
9	1.0055	1.0061	1.0068	1.0071
10	1.0056	1.0062	1.0069	1.0072
12	1.0057	1.0062	1.0069	1.0073
14	1.0057	1.0063	1.0069	1.0073
16	1.0057	1.0063	1.0069	1.0073
18	1.0057	1.0064	1.0069	1.0073
20	1.0057	1.0064	1.0068	1.0073
25	1.0057	1.0064	1.0068	1.0073
30	1.0057	1.0065	1.0068	1.0074
35	1.0057	1.0065	1.0068	1.0074
40	1.0058	1.0066	1.0068	1.0075
45	1.0058	1.0067	1.0068	1.0075
50	1.0059	1.0068	1.0068	1.0075
60	1.0060	1.0069	1.0069	1.0077
70	1.0061	1.0071	1.0070	1.0078
80	1.0062	1.0072	1.0071	1.0079
90	1.0063	1.0073	1.0072	1.0080
100	1.0064	1.0074	1.0073	1.0082
200	1.0074	1.0084	1.0081	1.0091

Table V.   Functions for calculating the higher approximations to

the transport coefficients. continued

The function $f_\lambda^{(3)}$				
T*	= 12	= 13	= 14	= 15
0	1.0027	1.0027	1.0027	1.0027
.1	1.0029	1.0028	1.0029	1.0028
.2	1.0027	1.0032	1.0033	1.0032
.3	1.0020	1.0023	1.0025	1.0026
.4	1.0006	1.0009	1.0011	1.0013
.5	1.0002	1.0002	1.0003	1.0005
.6	1.0002	1.0001	1.0001	1.0001
.7	1.0003	1.0001	1.0000	1.0000
.8	1.0003	1.0001	1.0000	1.0000
.9	1.0003	1.0001	1.0000	1.0000
1.0	1.0002	1.0001	1.0000	1.0000
1.2	1.0002	1.0001	1.0001	1.0002
1.4	1.0002	1.0003	1.0004	1.0005
1.6	1.0004	1.0006	1.0008	1.0010
1.8	1.0008	1.0011	1.0013	1.0016
2.0	1.0013	1.0016	1.0019	1.0022
2.5	1.0026	1.0031	1.0034	1.0039
3.0	1.0038	1.0045	1.0048	1.0054
3.5	1.0049	1.0056	1.0060	1.0067
4	1.0058	1.0065	1.0071	1.0077
5	1.0070	1.0078	1.0085	1.0091
6	1.0077	1.0085	1.0094	1.0099
7	1.0081	1.0090	1.0105	1.0105
8	1.0085	1.0093	1.0104	1.0109
9	1.0087	1.0095	1.0106	1.0111
10	1.0088	1.0097	1.0107	1.0112
12	1.0089	1.0098	1.0109	1.0114
14	1.0089	1.0099	1.0108	1.0114
16	1.0089	1.0099	1.0108	1.0115
18	1.0088	1.0100	1.0108	1.0115
20	1.0089	1.0100	1.0107	1.0114
25	1.0089	1.0101	1.0106	1.0115
30	1.0089	1.0101	1.0106	1.0115
35	1.0090	1.0102	1.0106	1.0116
40	1.0090	1.0103	1.0106	1.0116
45	1.0091	1.0104	1.0106	1.0117
50	1.0092	1.0105	1.0107	1.0118
60	1.0093	1.0108	1.0108	1.0120
70	1.0096	1.0111	1.0109	1.0122
80	1.0097	1.0112	1.0110	1.0124
90	1.0099	1.0114	1.0113	1.0125
100	1.0101	1.0116	1.0114	1.0127
200	1.0115	1.0132	1.0127	1.0141

Table V.   Functions for calculating the higher approximations to

the transport coefficients. continued

The function $f_D^{(3)}$				
T*	= 12	= 13	= 14	= 15
0	1.0020	1.0020	1.0020	1.0020
.1	1.0017	1.0017	1.0018	1.0018
.2	1.0010	1.0012	1.0013	1.0014
.3	1.0002	1.0004	1.0005	1.0006
.4	1.0001	1.0001	1.0001	1.0002
.5	1.0002	1.0001	1.0000	1.0000
.6	1.0003	1.0001	1.0000	1.0000
.7	1.0003	1.0001	1.0000	1.0000
.8	1.0002	1.0001	1.0000	1.0000
.9	1.0002	1.0001	1.0000	1.0000
1.0	1.0001	1.0000	1.0000	1.0001
1.2	1.0001	1.0001	1.0002	1.0003
1.4	1.0001	1.0003	1.0004	1.0006
1.6	1.0003	1.0005	1.0007	1.0009
1.8	1.0006	1.0008	1.0011	1.0013
2.0	1.0009	1.0012	1.0015	1.0017
2.5	1.0017	1.0020	1.0024	1.0028
3.0	1.0024	1.0028	1.0032	1.0037
3.5	1.0030	1.0035	1.0039	1.0044
4	1.0035	1.0041	1.0045	1.0050
5	1.0042	1.0049	1.0053	1.0059
6	1.0047	1.0053	1.0059	1.0064
7	1.0049	1.0057	1.0062	1.0068
8	1.0051	1.0059	1.0065	1.0070
9	1.0052	1.0060	1.0066	1.0072
10	1.0053	1.0061	1.0067	1.0073
12	1.0053	1.0061	1.0068	1.0074
14	1.0053	1.0062	1.0069	1.0075
16	1.0052	1.0061	1.0068	1.0075
18	1.0052	1.0061	1.0068	1.0075
20	1.0051	1.0060	1.0068	1.0075
25	1.0050	1.0059	1.0068	1.0075
30	1.0049	1.0058	1.0067	1.0075
35	1.0048	1.0058	1.0066	1.0075
40	1.0048	1.0058	1.0067	1.0075
45	1.0047	1.0058	1.0066	1.0075
50	1.0047	1.0058	1.0067	1.0076
60	1.0047	1.0058	1.0067	1.0077
70	1.0047	1.0058	1.0068	1.0077
80	1.0047	1.0059	1.0069	1.0078
90	1.0047	1.0059	1.0070	1.0079
100	1.0048	1.0060	1.0070	1.0081
200	1.0053	1.0067	1.0077	1.0088

Table VI.   The reduced thermal diffusion ratio, $\left[k_T{}^*\right]_2$, for calculating isotopic thermal diffusion.

$$k_T \cdot k_T^* \quad \frac{M_1 - M_2}{M_1 + M_2} \quad x_1 x_2$$

T*	$\alpha = 12$	$\alpha = 13$	$\alpha = 14$	$\alpha = 15$
0	0.3133	0.3133	0.3133	0.3133
.1	0.2844	0.2865	0.2887	0.2887
.2	0.2047	0.2264	0.2401	0.2468
.3	0.0738	0.1110	0.1369	0.1545
.4	-0.0271	0.0157	0.0484	0.0727
.5	-0.0829	-0.0389	-0.0037	0.0229
.6	-0.1038	-0.0616	-0.0256	0.0020
.7	-0.1031	-0.0623	-0.0270	0.0005
.8	-0.0902	-0.0509	-0.0162	0.0105
.9	-0.0690	-0.0317	0.0023	0.0287
1.0	-0.0456	-0.0088	0.0244	0.0498
1.2	0.0071	0.0419	0.0734	0.0981
1.4	0.0580	0.0923	0.1226	0.1466
1.6	0.1050	0.1386	0.1680	0.1911
1.8	0.1476	0.1797	0.2083	0.2310
2.0	0.1841	0.2161	0.2438	0.2665
2.5	0.2570	0.2883	0.3145	0.3380
3.0	0.3103	0.3404	0.3658	0.3890
3.5	0.3469	0.3786	0.4035	0.4270
4	0.3744	0.4060	0.4313	0.4544
5	0.4111	0.4437	0.4679	0.4931
6	0.4312	0.4643	0.4911	0.5134
7	0.4414	0.4778	0.5044	0.5283
8	0.4494	0.4858	0.5131	0.5358
9	0.4531	0.4889	0.5181	0.5427
10	0.4536	0.4911	0.5208	0.5446
12	0.4536	0.4917	0.5233	0.5494
14	0.4512	0.4919	0.5225	0.5495
16	0.4469	0.4887	0.5209	0.5496
18	0.4441	0.4866	0.5187	0.5485
20	0.4387	0.4826	0.5190	0.5479
25	0.4321	0.4764	0.5140	0.5452
30	0.4245	0.4713	0.5106	0.5427
35	0.4188	0.4681	0.5066	0.5429
40	0.4161	0.4648	0.5058	0.5426
45	0.4118	0.4641	0.5038	0.5426
50	0.4093	0.4616	0.5035	0.5429
60	0.4051	0.4598	0.5032	0.5443
70	0.4048	0.4590	0.5044	0.5460
80	0.4034	0.4583	0.5071	0.5474
90	0.4022	0.4584	0.5093	0.5505
100	0.4024	0.4586	0.5095	0.5534
200	0.4110	0.4703	0.5277	0.5727

Table VII.   Functions for calculating the transport properties

of mixtures.

The function A^*				
T^*	$\alpha = 12$	$\alpha = 13$	$\alpha = 14$	$\alpha = 15$
0	1.0065	1.0065	1.0065	1.0065
.1	1.0060	1.0146	1.0247	1.0251
.2	1.0374	1.0374	1.0445	1.0435
.3	1.0731	1.0693	1.0729	1.0695
.4	1.0976	1.0932	1.0946	1.0905
.5	1.1108	1.1067	1.1070	1.1028
.6	1.1170	1.1131	1.1128	1.1088
.7	1.1193	1.1154	1.1145	1.1108
.8	1.1194	1.1154	1.1142	1.1104
.9	1.1185	1.1143	1.1126	1.1091
1.0	1.1173	1.1126	1.1106	1.1071
1.2	1.1146	1.1096	1.1067	1.1032
1.4	1.1126	1.1070	1.1037	1.1000
1.6	1.1114	1.1052	1.1013	1.0978
1.8	1.1111	1.1044	1.0999	1.0964
2.0	1.1111	1.1044	1.0990	1.0956
2.5	1.1130	1.1056	1.0987	1.0958
3.0	1.1161	1.1081	1.1002	1.0973
3.5	1.1193	1.1113	1.1026	1.0995
4	1.1226	1.1141	1.1050	1.1018
5	1.1284	1.1194	1.1101	1.1064
6	1.1335	1.1238	1.1147	1.1103
7	1.1377	1.1277	1.1189	1.1136
8	1.1417	1.1309	1.1224	1.1163
9	1.1449	1.1336	1.1255	1.1189
10	1.1477	1.1361	1.1281	1.1210
12	1.1532	1.1406	1.1326	1.1246
14	1.1583	1.1447	1.1366	1.1276
16	1.1625	1.1485	1.1397	1.1300
18	1.1665	1.1517	1.1423	1.1322
20	1.1702	1.1548	1.1449	1.1343
25	1.1790	1.1623	1.1502	1.1385
30	1.1865	1.1691	1.1542	1.1421
35	1.1938	1.1752	1.1578	1.1454
40	1.2005	1.1813	1.1612	1.1480
45	1.2067	1.1867	1.1643	1.1508
50	1.2123	1.1917	1.1668	1.1531
60	1.2230	1.2008	1.1715	1.1573
70	1.2326	1.2092	1.1759	1.1610
80	1.2416	1.2168	1.1796	1.1642
90	1.2492	1.2240	1.1831	1.1674
100	1.2569	1.2300	1.1858	1.1700
200	1.3103	1.2754	1.2068	1.1885

Table VII. Functions for calculating the transport properties

of mixtures. continued.

The function B^*				
T^*	$\alpha = 12$	$\alpha = 13$	$\alpha = 14$	$\alpha = 15$
0	1.1851	1.1851	1.1851	1.1851
.1	1.1988	1.1955	1.1926	1.1928
.2	1.2528	1.2407	1.2328	1.2268
.3	1.3028	1.2895	1.2788	1.2698
.4	1.3127	1.3019	1.2922	1.2840
.5	1.3003	1.2922	1.2844	1.2768
.6	1.2808	1.2737	1.2668	1.2609
.7	1.2601	1.2539	1.2476	1.2424
.8	1.2403	1.2346	1.2289	1.2238
.9	1.2239	1.2172	1.2125	1.2074
1.0	1.2079	1.2025	1.1973	1.1930
1.2	1.1837	1.1780	1.1733	1.1689
1.4	1.1662	1.1612	1.1561	1.1523
1.6	1.1536	1.1490	1.1440	1.1392
1.8	1.1458	1.1394	1.1347	1.1298
2.0	1.1387	1.1324	1.1279	1.1226
2.5	1.1288	1.1222	1.1174	1.1119
3.0	1.1250	1.1167	1.1113	1.1057
3.5	1.1217	1.1136	1.1084	1.1024
4	1.1211	1.1128	1.1067	1.1002
5	1.1225	1.1126	1.1048	1.1001
6	1.1242	1.1131	1.1059	1.0989
7	1.1252	1.1155	1.1067	1.1006
8	1.1278	1.1176	1.1079	1.1009
9	1.1298	1.1185	1.1094	1.1025
10	1.1317	1.1203	1.1103	1.1026
12	1.1351	1.1225	1.1127	1.1050
14	1.1376	1.1247	1.1135	1.1056
16	1.1384	1.1259	1.1149	1.1066
18	1.1405	1.1271	1.1153	1.1070
20	1.1404	1.1274	1.1178	1.1077
25	1.1426	1.1282	1.1176	1.1070
30	1.1426	1.1281	1.1178	1.1058
35	1.1427	1.1279	1.1162	1.1055
40	1.1427	1.1268	1.1159	1.1050
45	1.1418	1.1270	1.1148	1.1035
50	1.1407	1.1255	1.1142	1.1023
60	1.1388	1.1236	1.1119	1.1003
70	1.1378	1.1218	1.1106	1.0980
80	1.1365	1.1198	1.1096	1.0950
90	1.1350	1.1180	1.1077	1.0940
100	1.1331	1.1154	1.1049	1.0922
200	1.1206	1.1021	1.0911	1.0779

Table VII. Functions for calculating the transport properties of mixtures. continued.

	The function C*			
T*	$\alpha = 12$	$\alpha = 13$	$\alpha = 14$	$\alpha = 15$
0	0.8889	0.8889	0.8889	0.8889
.1	0.8835	0.8844	0.8853	0.8853
.2	0.8697	0.8736	0.8765	0.8779
.3	0.8463	0.8531	0.8581	0.8613
.4	0.8279	0.8360	0.8421	0.8467
.5	0.8172	0.8257	0.8325	0.8376
.6	0.8128	0.8212	0.8283	0.8337
.7	0.8127	0.8209	0.8280	0.8334
.8	0.8151	0.8231	0.8301	0.8354
.9	0.8193	0.8269	0.8338	0.8391
1.0	0.8240	0.8315	0.8383	0.8434
1.2	0.8348	0.8419	0.8482	0.8532
1.4	0.8453	0.8521	0.8582	0.8629
1.6	0.8549	0.8615	0.8673	0.8718
1.8	0.8636	0.8698	0.8753	0.8797
2.0	0.8709	0.8771	0.8823	0.8867
2.5	0.8855	0.8913	0.8961	0.9005
3.0	0.8959	0.9014	0.9059	0.9102
3.5	0.9031	0.9088	0.9131	0.9174
4	0.9084	0.9141	0.9183	0.9226
5	0.9155	0.9213	0.9253	0.9298
6	0.9195	0.9254	0.9298	0.9337
7	0.9216	0.9280	0.9324	0.9365
8	0.9232	0.9295	0.9342	0.9381
9	0.9241	0.9302	0.9352	0.9394
10	0.9242	0.9307	0.9359	0.9399
12	0.9245	0.9311	0.9366	0.9409
14	0.9243	0.9313	0.9367	0.9412
16	0.9237	0.9310	0.9366	0.9414
18	0.9234	0.9307	0.9364	0.9413
20	0.9226	0.9302	0.9366	0.9414
25	0.9218	0.9295	0.9361	0.9413
30	0.9209	0.9291	0.9358	0.9412
35	0.9203	0.9289	0.9354	0.9415
40	0.9202	0.9288	0.9355	0.9417
45	0.9198	0.9290	0.9355	0.9420
50	0.9197	0.9290	0.9356	0.9423
60	0.9196	0.9293	0.9360	0.9430
70	0.9202	0.9298	0.9367	0.9437
80	0.9205	0.9303	0.9375	0.9444
90	0.9208	0.9309	0.9383	0.9453
100	0.9214	0.9316	0.9387	0.9461
200	0.9271	0.9377	0.9445	0.9520

Table VIIIA. Temperature indexes for viscosity, thermal conductivity,

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and self-diffusion.

T^* \ α	Index for η and λ				Index for D_{11}			
	12	13	14	15	12	13	14	15
0	0.833	0.833	0.833	0.833	0.833	0.833	0.833	0.833
.1	0.827	0.831	0.830	0.830	0.850	0.847	0.844	0.844
.2	0.832	0.819	0.818	0.820	0.891	0.879	0.870	0.866
.3	0.874	0.858	0.851	0.846	0.961	0.941	0.926	0.916
.4	0.950	0.925	0.912	0.399	1.016	0.992	0.974	0.960
.5	1.007	0.980	0.964	0.947	1.049	1.023	1.003	0.987
.6	1.040	1.014	0.997	0.979	1.062	1.036	1.015	0.999
.7	1.056	1.032	1.013	0.996	1.062	1.037	1.016	1.000
.8	1.058	1.036	1.018	1.001	1.055	1.031	1.010	0.994
.9	1.052	1.031	1.013	0.997	1.042	1.019	0.999	0.983
1.0	1.040	1.020	1.003	0.988	1.028	1.005	0.985	0.970
1.2	1.009	0.991	0.975	0.960	0.996	0.974	0.955	0.941
1.4	0.974	0.957	0.943	0.928	0.964	0.944	0.925	0.911
1.6	0.941	0.924	0.912	0.897	0.935	0.915	0.898	0.885
1.8	0.911	0.894	0.884	0.869	0.909	0.891	0.874	0.861
2.0	0.884	0.868	0.858	0.844	0.887	0.869	0.853	0.840
2.5	0.832	0.816	0.807	0.793	0.844	0.826	0.812	0.799
3.0	0.796	0.780	0.771	0.758	0.812	0.796	0.782	0.769
3.5	0.770	0.755	0.746	0.733	0.791	0.774	0.761	0.748
4	0.753	0.738	0.726	0.715	0.775	0.758	0.745	0.732
5	0.730	0.715	0.701	0.692	0.753	0.736	0.724	0.711
6	0.717	0.703	0.687	0.679	0.742	0.724	0.711	0.699
7	0.710	0.695	0.678	0.671	0.735	0.716	0.703	0.690
8	0.706	0.691	0.674	0.666	0.730	0.711	0.697	0.686
9	0.703	0.688	0.672	0.664	0.728	0.709	0.694	0.682
10	0.701	0.686	0.670	0.662	0.727	0.708	0.692	0.680
12	0.700	0.685	0.669	0.660	0.727	0.707	0.690	0.677
14	0.700	0.683	0.669	0.660	0.727	0.706	0.690	0.676
16	0.700	0.683	0.669	0.659	0.729	0.707	0.690	0.676
18	0.700	0.682	0.670	0.659	0.730	0.708	0.691	0.676
20	0.700	0.681	0.671	0.660	0.732	0.710	0.691	0.676
25	0.700	0.680	0.672	0.659	0.735	0.711	0.692	0.676
30	0.699	0.679	0.673	0.659	0.737	0.713	0.693	0.676
35	0.698	0.678	0.672	0.658	0.739	0.713	0.694	0.676
40	0.697	0.676	0.672	0.656	0.740	0.714	0.693	0.675
45	0.696	0.674	0.672	0.655	0.741	0.713	0.694	0.674
50	0.694	0.672	0.671	0.654	0.741	0.713	0.693	0.673
60	0.692	0.668	0.669	0.651	0.741	0.712	0.692	0.671
70	0.687	0.664	0.667	0.647	0.739	0.710	0.690	0.669
80	0.685	0.661	0.665	0.645	0.739	0.709	0.688	0.667
90	0.682	0.659	0.661	0.643	0.738	0.707	0.685	0.664
100	0.679	0.655	0.659	0.640	0.736	0.705	0.684	0.662
200	0.657	0.633	0.640	0.620	0.719	0.687	0.667	0.644

Table VIIIB. Sutherland constants for viscosity, thermal conductivity,

and self-diffusion.

T* \ α	S/T for η and λ				S/T for D_{11}			
	12	13	14	15	12	13	14	15
0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
.1	0.487	0.494	0.491	0.492	0.537	0.531	0.525	0.524
.2	0.497	0.469	0.467	0.471	0.642	0.611	0.588	0.578
.3	0.598	0.558	0.541	0.528	0.856	0.788	0.742	0.713
.4	0.819	0.739	0.701	0.664	1.067	0.969	0.900	0.852
.5	1.028	0.922	0.865	0.810	1.215	1.096	1.010	0.950
.6	1.176	1.059	0.988	0.921	1.281	1.157	1.063	0.996
.7	1.251	1.135	1.054	0.985	1.283	1.161	1.066	0.999
.8	1.264	1.154	1.073	1.003	1.246	1.131	1.040	0.975
.9	1.233	1.133	1.054	0.989	1.184	1.080	0.994	0.933
1.0	1.174	1.084	1.012	0.952	1.118	1.022	0.942	0.886
1.2	1.036	0.965	0.904	0.852	0.983	0.902	0.836	0.787
1.4	0.901	0.843	0.796	0.750	0.866	0.797	0.740	0.699
1.6	0.788	0.736	0.700	0.660	0.771	0.710	0.661	0.625
1.8	0.697	0.651	0.622	0.584	0.693	0.641	0.597	0.565
2.0	0.623	0.583	0.558	0.524	0.632	0.584	0.545	0.515
2.5	0.497	0.462	0.443	0.415	0.524	0.484	0.453	0.426
3.0	0.420	0.389	0.372	0.349	0.454	0.420	0.393	0.369
3.5	0.370	0.343	0.326	0.304	0.410	0.376	0.353	0.329
4	0.338	0.312	0.292	0.273	0.379	0.347	0.324	0.303
5	0.298	0.274	0.252	0.238	0.339	0.309	0.289	0.267
6	0.277	0.254	0.230	0.219	0.318	0.288	0.267	0.248
7	0.266	0.242	0.218	0.207	0.307	0.276	0.254	0.235
8	0.259	0.236	0.211	0.200	0.299	0.268	0.246	0.228
9	0.255	0.231	0.207	0.196	0.295	0.265	0.241	0.222
10	0.251	0.229	0.204	0.194	0.294	0.262	0.238	0.220
12	0.250	0.227	0.203	0.191	0.293	0.261	0.235	0.215
14	0.250	0.224	0.203	0.191	0.294	0.260	0.234	0.214
16	0.250	0.224	0.204	0.189	0.297	0.261	0.235	0.214
18	0.250	0.222	0.205	0.189	0.299	0.262	0.235	0.214
20	0.250	0.221	0.206	0.189	0.302	0.265	0.235	0.214
25	0.250	0.220	0.208	0.189	0.306	0.268	0.237	0.214
30	0.249	0.218	0.209	0.188	0.311	0.270	0.239	0.214
35	0.248	0.216	0.208	0.187	0.314	0.271	0.240	0.213
40	0.245	0.214	0.208	0.185	0.315	0.272	0.240	0.212
45	0.244	0.211	0.208	0.184	0.317	0.271	0.240	0.211
50	0.241	0.208	0.206	0.182	0.317	0.271	0.239	0.209
60	0.237	0.202	0.203	0.177	0.318	0.269	0.237	0.206
70	0.231	0.197	0.200	0.173	0.315	0.267	0.234	0.203
80	0.227	0.192	0.197	0.169	0.313	0.264	0.231	0.200
90	0.222	0.189	0.192	0.167	0.312	0.261	0.227	0.196
100	0.218	0.184	0.190	0.162	0.308	0.258	0.225	0.193
200	0.186	0.154	0.163	0.137	0.280	0.230	0.200	0.168
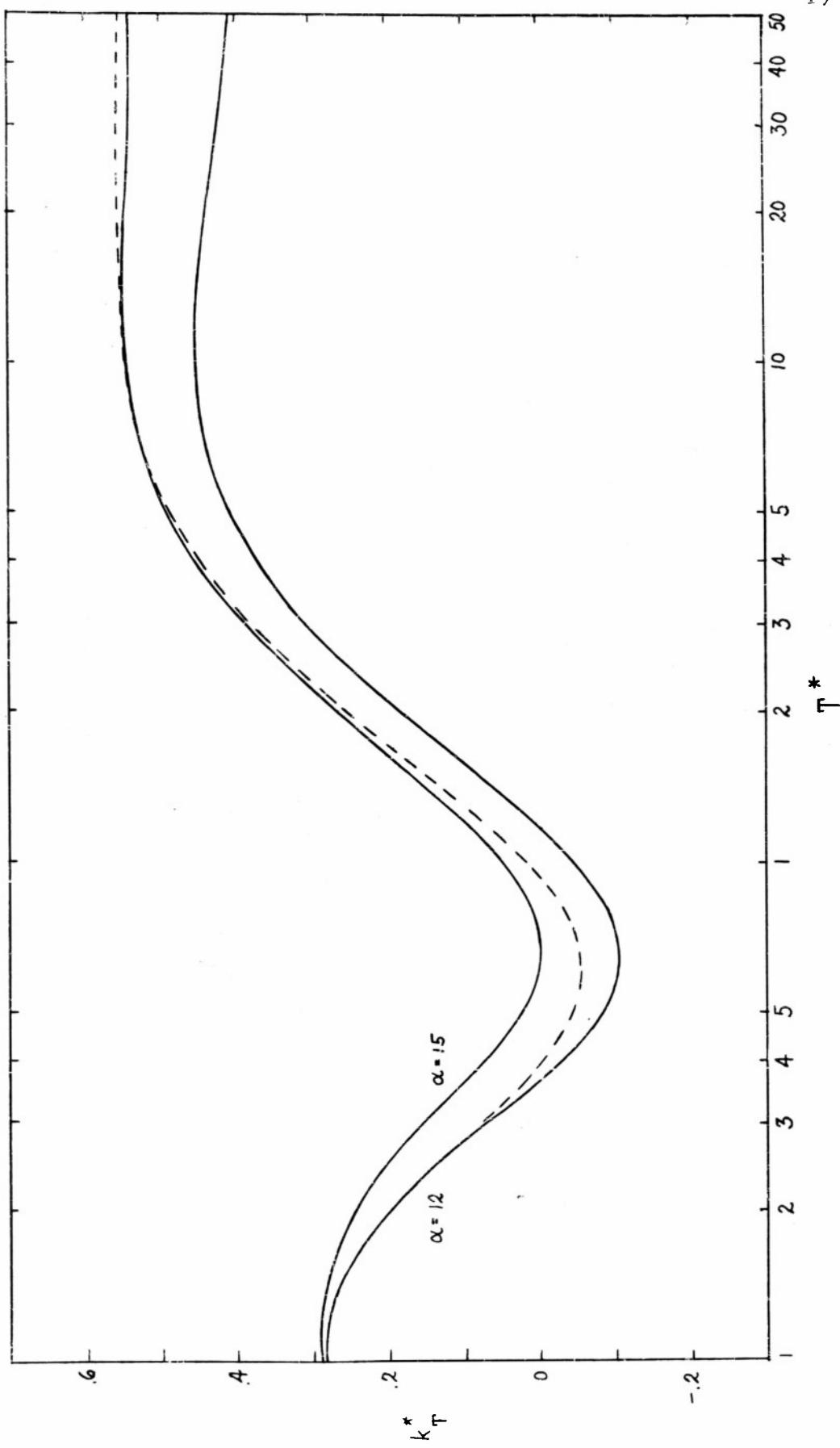
Fig. 3. The reduced thermal diffusion ratio, k_T^*, for isotopic thermal diffusion. Solid lines: second approximation for the Buckingham potential; broken line: first approximation for the Lennard-Jones (12-6) potential.
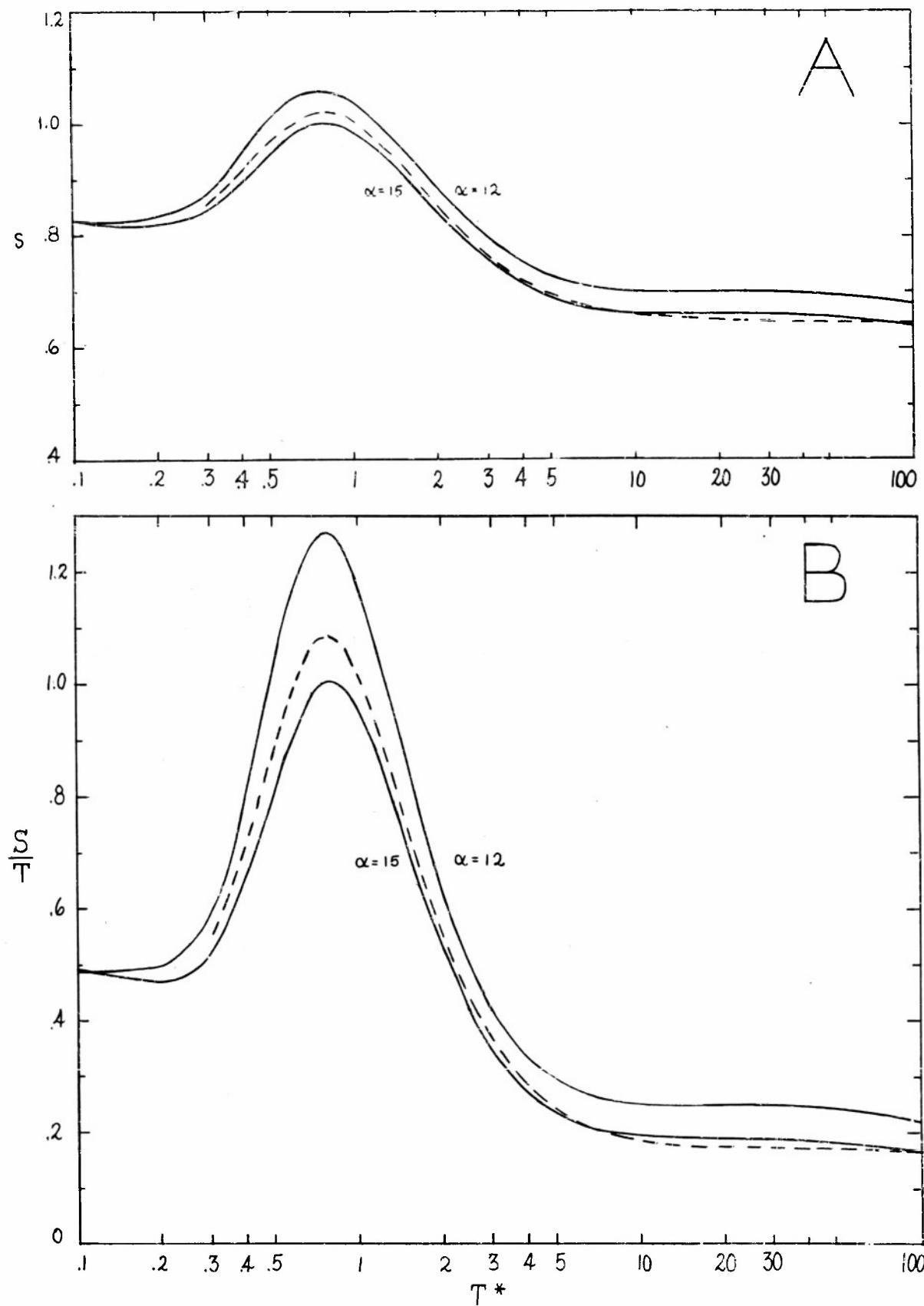
Fig. 4. A. Temperature indexes for viscosity and thermal conductivity;
B. Sutherland constants for viscosity and thermal conductivity. Solid
lines: Buckingham potential; broken lines: Lennard-Jones (12-6) potential.

ACKNOWLEDGMENTS

APPENDIX I. HIGHER APPROXIMATIONS TO THE TRANSPORT COEFFICIENTS

The quantity f_η appearing in Eq. (6) is given by the series

$$f_\eta = 1 + \gamma_1 + \gamma_2 + \cdots , \tag{35}$$

where

$$\gamma_1 = \frac{b_{12}^2}{b_{11} b_{22} - b_{12}^2} ,$$

$$\gamma_2 = \frac{b_{11} (b_{12} b_{23} - b_{22} b_{13})^2}{(b_{11} b_{22} - b_{12}^2)(b_{11} b_{22} b_{33} + 2 b_{12} b_{13} b_{23} - b_{11} b_{23}^2 - b_{22} b_{13}^2 - b_{33} b_{12}^2)} ;$$

and

$$b_{11} = 4 \, \Omega^{(2,2)*} ,$$

$$b_{12} = 7 \, \Omega^{(2,2)*} - 8 \Omega^{(2,3)*} ,$$

$$b_{13} = \frac{63}{8} \, \Omega^{(2,2)*} - 18 \Omega^{(2,3)*} + 10 \Omega^{(2,4)*} ,$$

$$b_{22} = \frac{301}{12} \, \Omega^{(2,2)*} - 28 \Omega^{(2,3)*} + 20 \Omega^{(2,4)*} ,$$

$$b_{23} = \frac{1365}{32} \, \Omega^{(2,2)*} - \frac{321}{4} \Omega^{(2,3)*} + \frac{125}{2} \Omega^{(2,4)*} - 30 \Omega^{(2,5)*} ,$$

$$b_{33} = \frac{25137}{256} \, \Omega^{(2,2)*} - \frac{1755}{8} \Omega^{(2,3)*} + \frac{1905}{8} \Omega^{(2,4)*} - 135 \Omega^{(2,5)*} + \frac{105}{2} \Omega^{(2,6)*} + 12 \Omega^{(4,4)*} .$$

The quantity f_λ appearing in Eq. (7) is given by the series

$$f_\lambda = 1 + \delta_1 + \delta_2 + \cdots , \tag{36}$$

where

$$\delta_1 = \frac{a''^2_{12}}{a''_{11} a''_{22} - a''^2_{12}}$$,

$$\delta_2 = \frac{a''_{11}(a''_{12} a''_{23} - a''_{22} a''_{13})^2}{(a''_{11} a''_{22} - a''^2_{12})(a''_{11} a''_{22} a''_{33} + 2 a''_{12} a''_{13} a''_{23} - a''_{11} a''^2_{23} - a''_{22} a''^2_{13} - a''_{33} a''^2_{12})}$$;

and

$$a''_{11} = b_{11} , \quad a''_{12} = b_{12} , \quad a''_{13} = b_{13} ;$$

$$a''_{22} = \frac{77}{4} \Omega^{(2,2)*} - 28 \Omega^{(2,3)*} + 20 \Omega^{(2,4)*} ,$$

$$a''_{23} = \frac{945}{32} \Omega^{(2,2)*} - \frac{261}{4} \Omega^{(2,3)*} + \frac{125}{2} \Omega^{(2,4)*} - 30 \Omega^{(2,5)*} ,$$

$$a''_{33} = \frac{14553}{256} \Omega^{(2,2)*} - \frac{1215}{8} \Omega^{(2,3)*} + \frac{1565}{8} \Omega^{(2,4)*} - 135 \Omega^{(2,5)*} + \frac{105}{2} \Omega^{(2,6)*} + 4 \Omega^{(4,4)*} .$$

The third approximation to the quantity f_D appearing in Eq. (8) is expressed in the determinant notation of Chapman and Cowling as

$$f_D^{(3)} = a_{00} \frac{\mathscr{A}^{(2)}_{00}}{\mathscr{A}^{(2)}} , \qquad (37)$$

where $\mathscr{A}^{(2)}$ is a fifth order determinant whose general element is a_{ij}, with i and j ranging from -2 to 2. The term $\mathscr{A}^{(2)}_{00}$ is the minor of $\mathscr{A}^{(2)}$ obtained by deleting the row and column containing a_{00}. The elements a_{ij} are functions of the collision integrals, the molecular weights, and the relative concentrations of the gases in the diffusing mixture. In general, $a_{ij} = a_{ji}$; and if $i \cdot j \neq 0$, a_{-i-j} has the same form as a_{ij} except that the

subscripts 1 and 2 , referring to the molecular species, are interchanged

in the expression for a_{ij} . The a_{ij} necessary for the evaluation of Eq. (37)

are as follows, where x_1 and x_2 are the mole fractions, and M_1 and M_2

the molecular weights of species 1 and 2 , respectively:

$$a_{00} = 8 \frac{M_1 M_2}{(M_1 + M_2)^2} \; \Omega_{12}^{(1,1)*} \qquad ;$$

$$a_{01} = 8 \frac{(2M_1)^{\frac{1}{2}} M_2^2}{(M_1 + M_2)^{\frac{5}{2}}} \left[\frac{5}{4} \Omega_{12}^{(1,1)*} - \frac{3}{2} \Omega_{12}^{(1,2)*} \right] \; ;$$

$$a_{02} = 4 \frac{(2M_1)^{\frac{1}{2}} M_2^3}{(M_1 + M_2)^{\frac{7}{2}}} \left[\frac{35}{8} \Omega_{12}^{(1,1)*} - \frac{21}{2} \Omega_{12}^{(1,2)*} + 6 \Omega_{12}^{(1,3)*} \right] \; ;$$

$$a_{0-1} = - \left(\frac{M_1}{M_2} \right)^{\frac{3}{2}} a_{01} \quad ; \quad a_{0-2} = - \left(\frac{M_1}{M_2} \right)^{\frac{5}{2}} a_{02} \; ;$$

$$a_{11} = a'_{11} + \frac{x_1}{x_2} a''_{11} , \tag{38}$$

$$a'_{11} = \frac{8M_2}{(M_1 + M_2)^3} \left[\frac{5}{8} (6M_1^2 + 5M_2^2) \Omega_{12}^{(1,1)*} - \frac{15}{2} M_2^2 \Omega_{12}^{(1,2)*} + 6 M_2^2 \Omega_{12}^{(1,3)*} + 2 M_1 M_2 \Omega_{12}^{(2,2)*} \right] ,$$

$$a''_{11} = 4 \, \Omega_{11}^{(2,2)*} \left(\frac{2M_2}{M_1 + M_2} \right)^{\frac{1}{2}} \left[\frac{(r_m)_{11}}{(r_m)_{12}} \right]^2 ;$$

$$a_{12} = a'_{12} + \frac{x_1}{x_2} a''_{12} \; ;$$

$$a'_{12} = \frac{8 M_2^2}{(M_1 + M_2)^4} \left[\begin{array}{l} \frac{35}{32}(12 M_1^2 + 5 M_2^2)\Omega_{12}^{(1,1)*} - \frac{63}{16}(4M_1^2 + 5M_2^2)\Omega_{12}^{(1,2)*} + \frac{57}{2} M_2^2 \Omega_{12}^{(1,3)*} \\[2mm] - 15 M_2^2 \Omega_{12}^{(1,4)*} + 7 M_1 M_2 \Omega_{12}^{(2,2)*} - 8 M_1 M_2 \Omega_{12}^{(2,3)*} \end{array} \right] ;$$

(38)
(cout.)

$$a''_{12} = \left(\frac{2M_2}{M_1 + M_2}\right)^{\frac{1}{2}} \left[7\Omega_{11}^{(2,2)*} - 8\Omega_{11}^{(2,3)*} \right]\left[\frac{(r_m)_{11}}{(r_m)_{12}} \right]^2 ;$$

$$a_{1-1} = - \frac{8 (M_1 M_2)^{\frac{3}{2}}}{(M_1 + M_2)^3} \left[\frac{55}{8} \Omega_{12}^{(1,1)*} - \frac{15}{2} \Omega_{12}^{(1,2)*} + 6 \Omega_{12}^{(1,3)*} - 2\Omega_{12}^{(2,2)*} \right] ;$$

$$a_{1-2} = - \frac{8 M_1^{\frac{5}{2}} M_2^{\frac{3}{2}}}{(M_1 + M_2)^4} \left[\begin{array}{l} \frac{595}{32} \Omega_{12}^{(1,1)*} - \frac{567}{16} \Omega_{12}^{(1,2)*} + \frac{57}{2} \Omega_{12}^{(1,3)*} - 15\Omega_{12}^{(1,4)*} \\[2mm] - 7\Omega_{12}^{(2,2)*} + 8\Omega_{12}^{(2,3)*} \end{array} \right] ;$$

$$a_{22} = a'_{22} + \frac{x_1}{x_2} a''_{22} \; ;$$

$$a'_{22} = \frac{8 M_2}{(M_1 + M_2)^5} \left[\begin{array}{l} \frac{35}{128}(40M_1^4 + 168 M_1^2 M_2^2 + 35 M_2^4)\Omega_{12}^{(1,1)*} - \frac{21 M_2^2}{16}(84 M_1^2 + 35 M_2^2)\Omega_{12}^{(1,2)*} \\[2mm] + \frac{3}{4} M_2^2 (108 M_1^2 + 133 M_2^2)\Omega_{12}^{(1,3)*} - 105 M_2^4 \Omega_{12}^{(1,4)*} + 45 M_2^4 \Omega_{12}^{(1,5)*} \\[2mm] + \frac{7}{2} M_1 M_2 (4M_1^2 + 7M_2^2)\Omega_{12}^{(2,2)*} - 56 M_1 M_2^3 \Omega_{12}^{(2,3)*} + 40 M_1 M_2^3 \Omega_{12}^{(2,4)*} \\[2mm] + 12 M_1^2 M_2^2 \Omega_{12}^{(3,3)*} \end{array} \right] ;$$

$$a''_{22} = \left(\frac{2M_2}{M_1 + M_2}\right)^{\frac{1}{2}} \left[\frac{77}{4} \Omega_{11}^{(2,2)*} - 28\Omega_{11}^{(2,3)*} + 20\Omega_{11}^{(2,4)*} \right]\left[\frac{(r_m)_{11}}{(r_m)_{12}} \right]^2 ;$$

$$a_{2-2} = - \frac{8(M_1 M_2)^{\frac{5}{2}}}{(M_1 + M_2)^5} \left[\begin{array}{l} \frac{8505}{128} \Omega_{12}^{(1,1)*} - \frac{2499}{16}\Omega_{12}^{(1,2)*} + \frac{723}{4} \Omega_{12}^{(1,3)*} - 105 \Omega_{12}^{(1,4)*} + 45 \Omega_{12}^{(1,5)*} \\[2mm] - \frac{77}{2} \Omega_{12}^{(2,2)*} + 56\Omega_{12}^{(2,3)*} - 40 \Omega_{12}^{(2,4)*} + 12\Omega_{12}^{(3,3)*} \end{array} \right] .$$

The a'_{ij} above are the same as those defined in connection with the thermal conductivity, but now it is necessary to specify, by subscripts on the $\Omega^{(\ell, n)*}$, which molecular interaction is under consideration; for example, a''_{11} refers to an interaction between two molecules of species 1, whereas a''_{-1-1} refers to species 2. The above expression for the a_{ij} were calculated by the method of Chapman and Cowling[34].

The expression for f_D may be considerably simplified for the cases of tracer diffusion and self-diffusion. For tracer diffusion where $x_1 \longrightarrow 0$, we obtain

$$f_D = \frac{1}{1 - \triangle_1 - \triangle_2 - \cdots} \quad , \qquad (39)$$

where

$$\triangle_1 = \frac{a_{01}^2}{a_{00} \, a'_{11}} = \frac{M_2^2 (6C^* - 5)^2}{30 M_1^2 + 25 M_2^2 - 12 M_2^2 B^* + 16 M_1 M_2 A^*} \quad ,$$

$$\triangle_2 = \frac{(a_{01} \, a'_{12} - a_{02} \, a'_{11})^2}{a_{00} \, a'_{11} (a'_{11} \, a'_{22} - a'^2_{12})} \quad .$$

The quantities A^*, B^*, and C^* are defined in Eq. (11). The result for $x_2 \longrightarrow 0$ is obtained from the above by changing the signs of all subscripts on the a_{ij}, or what has the same effect, interchanging M_1 and M_2. The expression for self-diffusion is obtained by setting $M_1 = M_2$, $\Omega_{12}^{(\ell, n)*} = \Omega_{11}^{(\ell, n)*} = \Omega_{22}^{(\ell, n)*}$, and $(r_m)_{12} = (r_m)_{11} = (r_m)_{22}$.

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[34] Reference (4), Chapter 9.

The general expression for the thermal diffusion ratio given by Eq. (9) involves the a_{ij} defined above. For a binary mixture of heavy isotopes, the dimensionless quantity k_T^*, defined in Eq. (10) is given to a first approximation by

$$\left[k_T^* \right]_1 = \frac{15}{2} \left(\frac{a_{0-1}}{a'_{11} \, a''_{11}} \right) \left[5 \, \Omega^{(1,1)*} + 2 \Omega^{(2,2)*} \right]$$

$$= \frac{15}{2} \frac{(6C^* - 5)(2A^* + 5)}{A^* (16 A^* - 12 B^* + 55)} \quad , \tag{40}$$

and to a second approximation by

$$\left[k_T^* \right]_2 = \frac{5}{2} \left[X_1 - X_2 Y_1 - X_3 Y_2 \right] , \tag{41}$$

where

$$X_1 = \frac{3 a_{0-1} a''_{22} - 5 a_{0-2} a''_{12}}{a''_{11} a''_{22} - a''^2_{12}} \quad ,$$

$$X_2 = \frac{a_{0-1} a'_{22} - a_{0-2} a'_{12}}{a'_{11} a'_{22} - a'^2_{12}} \quad ,$$

$$X_3 = \frac{a_{0-2} a'_{11} - a_{0-1} a'_{12}}{a'_{11} a'_{22} - a'^2_{12}} \quad ,$$

$$Y_1 = \frac{c_1 a''_{22} - c_2 a''_{12}}{a''_{11} a''_{22} - a''^2_{12}} \quad ,$$

$$Y_2 = \frac{c_2 a''_{22} - c_3 a''_{12}}{a''_{11} a''_{22} - a''^2_{12}} \quad ,$$

and

$$c_1 = \frac{45}{8} \Omega^{(1,1)*} - \frac{45}{2} \Omega^{(1,2)*} + 18 \Omega^{(1,3)*} \quad,$$

$$c_2 = \frac{1295}{64} \Omega^{(1,1)*} - \frac{1827}{32} \Omega^{(1,2)*} + \frac{285}{4} \Omega^{(1,3)*} - \frac{75}{2} \Omega^{(1,4)*} \quad,$$

$$c_3 = \frac{7805}{512} \Omega^{(1,1)*} - \frac{5439}{64} \Omega^{(1,2)*} + \frac{2319}{16} \Omega^{(1,3)*} - \frac{525}{4} \Omega^{(1,4)*} + \frac{225}{4} \Omega^{(1,5)*}$$

$$- \frac{35}{8} \Omega^{(2,2)*} - 14 \Omega^{(2,3)*} + 10 \Omega^{(2,4)*} + 3 \Omega^{(3,3)*} \quad.$$

In the above expressions for k_T^* the a_{ij} are to be calculated as if $M_1 = M_2$, inasmuch as the deviations from this equality have been explicitly allowed for in deriving Eq. (10). It is interesting to note that for rigid spheres $\left[k_T^* \right]_2$ is 8.8 percent larger than $\left[k_T^* \right]_1$.

The above expressions for the higher approximations to the transport coefficients were calculated by the method of Chapman and Cowling[35], and their form depends on a particular approximation scheme for obtaining solutions to an infinite set of simultaneous equations. The scheme used by Chapman and Cowling is not unique, and indeed Kihara[36] has developed an alternative scheme and from it obtained the following second approximations to viscosity, thermal conductivity, and self-diffusion, and the first approximation to the thermal diffusion ratio for isotopic mixtures:

$$f_\eta^{(2)} = 1 + \frac{3}{49} \left[\frac{4 \Omega^{(2,3)*}}{\Omega^{(2,2)*}} - \frac{7}{2} \right]^2 \quad, \tag{42}$$

$$f_{\lambda}^{(2)} = 1 + \frac{2}{21}\left[\frac{4\,\Omega^{(2,3)*}}{\Omega^{(2,2)*}} - \frac{7}{2}\right]^2 \qquad , \tag{43}$$

$$f_{D}^{(2)} = 1 + \frac{(6C^* - 5)^2}{16A^* + 40} \qquad , \tag{44}$$

$$\left[k_T^*\right]_1 = \frac{15}{16}\,\frac{(6C^* - 5)}{A^*} \qquad . \tag{45}$$

Kihara's expressions are considerably simpler than the corresponding

Chapman and Cowling expressions, and are known to be more accurate for

the special case of a Lorentzian gas (a binary mixture in which one component

has a much smaller molecular weight than the other, and collisions between

molecules of the light component need not be considered). It is, therefore,

of interest to compare the expressions of Kihara and of Chapman and Cowling

for the more realistic potential given by Eq. (1), and to estimate their

relative accuracy by comparison with the still higher approximations

evaluated in the present paper.

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(35) Reference (4), Chapter 7.

(36) T. Kihara, Imperfect Gases (originally published in Japanese in 1949, and translated into English by the U. S. Air Force), Secs. 23, 26, 27, 31, 32; "Virial Coefficients and Models of Molecules in Gases", University of Wisconsin Report OOR-7, June 5, 1953, to be submitted to Rev. Mod. Phys. for publication.

The two methods yield answers for the potential of Eq. (1) which are appreciably different only for the case of the thermal diffusion ratio; some selected values for isotopes are given in Table IX below. The overall accuracy of Kihara's expression seems to be somewhat better. In view of this and of their greater simplicity (fewer different collision integrals are required), Kihara's expressions for the second approximation to viscosity and thermal conductivity and the first approximation to the thermal diffusion ratio would seem to be definitely preferable to Chapman and Cowling's corresponding expressions. It is for this reason that we have avoided expressing our results for isotopic thermal diffusion in terms of the usual quantity R_T, defined as the ratio of k_T to the first approximation to k_T for rigid spheres. Since the first approximations to k_T for rigid spheres according to Chapman and Cowling and to Kihara are different, R_T involves an uncertain numerical factor, and the quantity k_T^* defined in Eq. (10) seems preferable from a theoretical standpoint.

Table IX. Comparison of the approximations of Chapman and Cowling (CC) and of Kihara (K) to the reduced thermal diffusion ratio for isotopes, k_T^* .

T*	$\left[k_T^*\right]_1$		$\left[k_T^*\right]_2$
	CC	K	CC
		$\alpha = 12$	
0	0.3062	0.3104	0.3133
0.5	-0.0828	-0.0819	-0.0829
1	-0.0465	-0.0469	-0.0456
2	0.1860	0.1903	0.1841
10	0.4349	0.4455	0.4536
20	0.4197	0.4291	0.4387
100	0.3852	0.3942	0.4024
		$\alpha = 15$	
0	0.3062	0.3104	0.3133
0.5	0.0217	0.0216	0.0229
1	0.0505	0.0511	0.0498
2	0.2668	0.2739	0.2665
10	0.5189	0.5348	0.5446
20	0.5204	0.5357	0.5479
100	0.5253	0.5422	0.5534

APPENDIX II. COMPARISON WITH AMDUR'S SEMI-CLASSICAL

CALCULATION OF $S^{(\ell)}(K)$

Amdur has devised an approximate method[37] for the calculation

of gaseous transport properties which is capable of handling very complicated

intermolecular potential functions with relatively little computation, and has

applied his method to the calculation of the transport properties of the rare

gases at low temperatures. The procedure, however, involves assumptions

which require further justification[38] , so that it is of interest to provide a

test of the assumptions by comparing the results of the present calculations

with those obtained by applying Amdur's method to the same intermolecular

potential.

In the approximate procedure the transport cross sections are not

calculated directly, but instead the total classical collision cross section is

first obtained, and the transport cross sections are then assumed to be in the

same ratio to the total collision cross section as in the case for rigid spheres.

The total collision cross section is infinite if calculated by completely classical

means, so a semi-classical approximation is used, in which only scattering

through angles greater than a certain critical angle, χ_c , is considered. The

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[37] I. Amdur, J. Chem. Phys. 15, 482 (1947); 16, 190 (1948); 17, 100
(1949).

[38] H. S. W. Massey and E. H. S. Burhop, Electronic and Ionic Impact
Phenomena (Oxford University Press, London, England, 1952), pp. 381-
382, 385.

total collision cross section, $S(K)$, is then given by

$$S(K) = \pi \, b_c^2 \, , \tag{46}$$

where b_c is that value of the collision parameter which, for a given value of the energy K , will produce a deflection equal to χ_c . The velocity dependent cross sections for viscosity and diffusion are then taken to be two-thirds and one-half, respectively, of $S(K)$, as for rigid spheres. The collision integrals are obtained from the cross sections by essentially the same procedure as used in the present paper.

Amdur's method was applied to the potential given in Eq. (1) to calculate approximate values of the velocity dependent transport cross sections for comparison with the accurate values obtained in this paper by direct means. The critical scattering angle was taken to be[39]

$$\chi_c = \frac{\Lambda}{2 y K^{\frac{1}{2}}} \, , \tag{47}$$

where Λ is the dimensionless group $h / \left[r_m (2 \mu \epsilon)^{1/2} \right]$, h is Planck's constant, and the other quantities are as previously defined. Following Amdur, χ_c was represented by Eq. (16) with the second term neglected; use of this equation restricts the range of the results to $K > 1$. The results are expressed in reduced form, $S^*(K) = S(K) / (\pi \, r_m^2)$. According to the approximation, $S^*(K) = S^{(l)}(K)$ for all values of l ; however for $K > 1$, it is actually found

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[39] H. S. W. Massey and C. B. O. Mohr, Proc. Roy Soc. (London) A141, 434 (1933).

that $S^{(2)}(K)$ is always larger than $S^{(1)}(K)$, by as much as 49 percent.

In Fig. 5, $S*(K)$ is shown for Λ = 0.05 and 2 , where the potential

parameter α = 12. The small value of Λ corresponds to a heavy gas with

strong intermolecular attraction, such as xenon; the large value to a light gas

with weak attraction, such as hydrogen. Also shown for comparison are

$S^{(1)}(K)$ and $S^{(2)}(K)$ for α = 12. The results for other values of α are quite

similar. It is seen that although $S*(K)$ does not change rapidly enough with

K , it may be nearly correct over a small range of K . This fact would

account for Amdur's numerical results on the rare gases (except helium),

where he found fair agreement between calculated and experimental values of

the viscosity, but the calculated temperature variation was too small. The

calculated values of the self-diffusion coefficients of neon and argon were,

however, rather too small[40][41], as might be expected since $S^{(1)}(K)$ is

really smaller than $S^{(2)}(K)$. The agreement for helium was somewhat better,

but this was perhaps due in part to the use of a potential which is no longer

considered very accurate[42].

The approximation scheme is thus seen to give results which are

valid over a limited range, and hence may still prove useful because of its

ability to handle very complicated potential functions.

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(40) E. B. Winn, Phys. Rev. 80, 1024 (1950).

(41) F. Hutchinson, J. Chem. Phys. 17, 1081 (1949).

(42) Reference (36), p. 378.
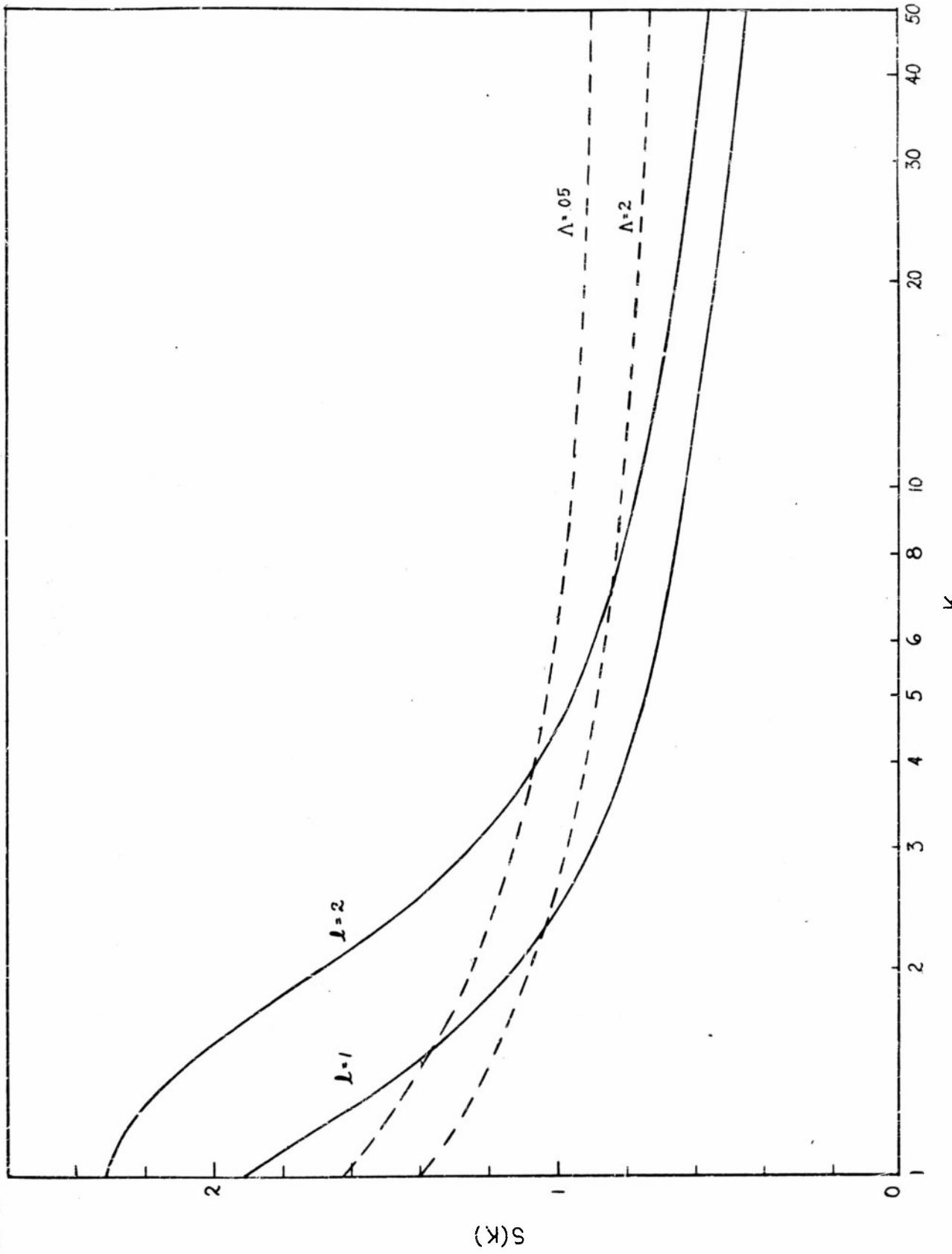
Fig. 5. Comparison of exact and approximate calculations of the reduced cross sections.
S(K), for the Buckingham potential. Solid lines: calculated by numerical integration;
broken lines: calculated by Amdur's approximation scheme.

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